65. On Weakly Compact Regular Spaces. I

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In their paper [3], S. Mardešić and P. Papić introduced a new kind of space, weakly compact topological space.

Let Φ be a family of subsets in topological space S. A point x of S is called *a point accumulation of* Φ, if every neighbourhood of x meets infinitely many members of Φ.

Definition. A topological space S is called *weakly compact*, if any family of disjoint non-empty open sets has at least one point of accumulation. S. Mardešić and P. Papić proved the following interesting proposition: *A regular space is weakly compact if and only if any open countable covering has the AU property in the sense of present writer.*

The proposition gives some elementary properties of a weakly compact regular space, which are analogous for the case of an absolute closed space (cf. M. Katětov [1]).

Theorem 1. A necessary and sufficient condition that a regular space S be weakly compact is that for every family of countable open sets G_n $(n=1, 2,\cdots)$ *of S having the finite intersection property the intersection of all* $\overline{G}_n$ *is non-empty.*

Proof. To prove the necessity of Theorem 1, let G_n be a given family of countable open sets having the finite intersection property. Suppose that $\bigcap_{n=1}^{\infty}\overline{G}_n=\phi$, then $S=S-\bigcap_{n=1}^{\infty}\overline{G}_n=\bigcup_{n=1}^{\infty}(S-\overline{G}_n)$ and $S-\overline{G}_n$ $(n=1, 2,\cdots)$ is an open covering of S. Hence we can find an index N such that $\bigcup_{n=1}^{N}\overline{S-\overline{G}_n}=S$.

Since each G_n is open, we have $\bigcap_{n=1}^{N}G_n=0$ which is a contradiction.

Conversely, let O_n $(n=1, 2,\cdots)$ be disjoint non-empty open sets. Let $G_n=\bigcup_{k=n}^{\infty}O_k$, then $\{G_n\}$ has the finite intersection property. Hence $\bigcap_{n=1}^{\infty}\overline{G}_n\neq\phi$. For p of $\bigcap_{n=1}^{\infty}G_n$, any neighbourhood $V(p)$ of p meets each G_n $(n=1, 2,\cdots)$. Therefore $V(p)$ meets infinitely many of O_n. Q.E.D.

Let S be a weakly compact regular space, and suppose that $\{F_n\}$ be a decreasing sequence of closed sets such that $\operatorname{Int}F_n\neq\phi$, where $\operatorname{Int}F$ is the interior of F. Then $\operatorname{Int}F_n$ are non-empty open sets having the finite intersection property, and $F_n\supset\overline{\operatorname{Int}F_n}$ for each n.