85. On the Completion of the Ranked Spaces

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1. In this note we shall consider the problem of completion:[1] construction of a complete ranked space[2] containing a given ranked space as a dense subspace.

Definition 1. Let R be a ranked space.[3] For a family F of fundamental sequences of R, we shall call a coordinate of F every neighbourhood $v(p)$ which is the first term of a fundamental sequence belonging to F. For two families F, G of fundamental sequences, $F\geq G$ means that, for every coordinate $v(p)$ of F, there exists a coordinate $u(q)$ of G such that $v(p)\supseteq u(q)$.

Definition 2. For a point p of R, $\mathfrak{S}(p)$ denotes the set of all fundamental sequences $u=\{u_\alpha(p_\alpha)\}$ such that $p_\alpha\equiv p$. Let R^* be the family of all families p^* of fundamental sequences satisfying the following conditions:

(1) If $u_\alpha=\{u^\alpha_\beta(p^\alpha_\beta);\ 0\leq\beta<\omega_{\mu_\alpha}\}$ $(0\leq\alpha<\gamma<\omega_\nu)$ belongs to p^* and $\lambda_\alpha(0\leq\lambda_\alpha<\omega_{\mu_\alpha})$ is an ordinal number, then there exists a member $u=\{u_\beta(p_\beta)\}$ of p^* such that $u_0(p_0)\subseteq\bigcap_\alpha u^\alpha_{\lambda_\alpha}(p^\alpha_{\lambda_\alpha})$.

(2) If $u=\{u_\alpha(p_\alpha)\}$, $v=\{v_\beta(q_\beta)\}\in p^*$, $u_0(p_0)\in\mathfrak{B}_{\gamma_0}$, $v_0(q_0)\in\mathfrak{B}_{\gamma'_0}$, $\gamma_0<\gamma'_0$ and $u_0(p_0)\supseteq v_0(q_0)$, then there exist a rank γ and $u(p_0)$ of rank γ such that $\gamma_0<\gamma\leq\gamma'$ and $u_0(p_0)\supseteq u(p_0)\supseteq v_0(q_0)$.

(3) $p^*\not\geq\mathfrak{S}(p)$ for any p except the case $p^*=\mathfrak{S}(p)$.

Then we obtain easily the following

Lemma 1. $\mathfrak{S}(p)$ satisfies the conditions (1) and (2). And, for an ω_ν-fundamental sequence $v=\{v_\alpha(p_\alpha);\ 0\leq\alpha<\omega_\nu\}$, let v^β denote the fundamental sequence $\{v_\alpha(p_\alpha);\ \beta\leq\alpha<\omega_\nu\}$ and v^* the set of such v^β, $0\leq\beta<\omega_\nu$. Then v^* satisfies the conditions (1) and (2).

Definition 3. For two members p^*, q^* of R^*, put $p^*\approx q^*$ if $p^*\geq q^*$ and $p^*\leq q^*$. By this equivalence relation, we shall classify R^* and denote this classification by $\hat{R}$. Let $W(V,\hat{p})$, where $\hat{p}$ is a point of $\hat{R}$ and V is a coordinate of some p^* belonging to $\hat{p}$, denote the set of all

1) Prof. K. Kunugi studied this problem in the notes "Sur les espaces complets et régulièrement complets. I–III", Proc. Japan Acad., **30**, 553–556, 912–916 (1954); **31**, 49–53 (1955).

2) See, for the notions and the terminologies, K. Kunugi, I., *Op. cit.*, H. Okano: Some operations on the ranked spaces. I, Proc. Japan Acad., **33**, 172–176 (1957) and H. Okano: On closed subspaces of the complete ranked spaces, Proc. Japan Acad., **33**, 336–337 (1957).

3) The rank of R is given by ω_ν. See K. Kunugi, I., *Op. cit.*