81. On Closed Mappings. II

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1. A topological space is said to be locally peripherally compact or semicompact (=semibicompact) if every point has arbitrarily small open neighbourhoods with compact boundaries. The purpose of this note is to establish the following theorems.

Theorem 1. *Let f be a quasi-compact continuous mapping of a locally peripherally compact Hausdorff space X onto a Hausdorff space Y such that, for each point y of Y, the inverse image $f^{-1}(y)$ is connected and the boundary $\mathfrak{B}f^{-1}(y)$ of $f^{-1}(y)$ is compact. Then f is a closed mapping and Y is locally peripherally compact.*

Theorem 2. *Let f be a closed continuous mapping of a locally peripherally compact Hausdorff space X onto a locally peripherally compact Hausdorff space Y such that $\mathfrak{B}f^{-1}(y)$ is compact for each point y of Y. Then f can be extended to a continuous mapping of $\gamma(X)$ onto $\gamma(Y)$, where $\gamma(X)$ and $\gamma(Y)$ mean the Freudenthal compactifications of X and Y respectively.*[*)]

Our Theorem 1 generalizes a theorem of A. H. Stone [6, Theorem 2] as well as a theorem of S. Hanai [2, Theorem 3].

2. **Proof of Theorem 1.** Let X be a locally peripherally compact Hausdorff space. A finite open covering $\{G_1,\cdots,G_r\}$ of X is called a γ-covering of X if $\mathfrak{B}G_i$ is compact for each i. Let $\{\mathfrak{U}_\lambda \mid \lambda\in\Lambda\}$ be the totality of all the γ-coverings of X. Then the following propositions are proved in our previous paper [3].

(1) For any two γ-coverings $\mathfrak{U}_\lambda$ and $\mathfrak{U}_\mu$ there exists a γ-covering $\mathfrak{U}_\nu$ which is a refinement of $\mathfrak{U}_\lambda$ and $\mathfrak{U}_\mu$.

(2) For any γ-covering $\mathfrak{U}_\lambda$ there exists a γ-covering $\mathfrak{U}_\mu$ which is a star-refinement of $\mathfrak{U}_\lambda$.

(3) For each point x of X, $\{S(x,\mathfrak{U}_\lambda)\mid\lambda\in\Lambda\}$ is a basis of neighbourhoods of x.

Now let f be a quasi-compact continuous mapping of X onto a Hausdorff space Y such that, for each point y of Y, $f^{-1}(y)$ is connected and $\mathfrak{B}f^{-1}(y)$ is compact. Let y_0 be any point of Y and let G be any open set of X containing $f^{-1}(y_0)$. Since $\mathfrak{B}f^{-1}(y_0)$ is compact and X is locally peripherally compact, there exist a finite number of open sets H_i, $i=1,\cdots,m$, of X such that $\mathfrak{B}H_i$ is compact and $H_i\subset G$ for each i, and that $\mathfrak{B}f^{-1}(y_0)\subset\smile\{H_i\mid i=1,\cdots,m\}$. Let $G_0=[\smile\{H_i\mid$

*) As for the Freudenthal compactifications, cf. [3].