78. *Inferior Limit of a Sequence of Potentials*

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1. In a locally compact space Ω we consider a sequence of potentials of positive measures. In case that Ω is the τ-dimensional euclidean space R^τ ($\tau \geqq 2$), a fundamental theorem, which was proved by Brelot [1], asserts that the inferior limit of a sequence of newtonian potentials is equal to a potential of a positive measure in the complement of a exceptional set E of inner capacity zero. Cartan [4], using the energy principle, showed that the set E is of outer capacity zero. Recently Brelot has proved that this fact follows from Choquet's result [5] on the capacitability of Borel sets. The problem of capacitability in the potential theory in a locally compact space has not yet been solved, and so in this note we shall prove under an additional condition that E is of outer capacity zero (see Brelot and Choquet [3]).

2. Let Ω be a locally compact space. We consider always positive measures μ in Ω with compact carrier, denoted by S_μ. Let $\Phi(P, Q)$ be a positive, symmetric, continuous and real valued function defined on $\Omega\times\Omega$, which is finite except at the points of the diagonal set of $\Omega\times\Omega$. The potential of μ is defined by

$$U^\mu(P)=\int\Phi(P, Q)\,d\mu(Q).$$

In this paper μ will be called *admissible* on a compact set K, if $S_\mu\subset K$ and $U^\mu(P)\leqq 1$ everywhere in Ω. The supremum of the total masses of admissible measures on K is defined to be the capacity of K and denoted by $\mathrm{cap}\,(K)$. The inner capacity $\mathrm{cap}_i\,(A)$ of $A\subset\Omega$ is equal to $\sup \mathrm{cap}\,(K)$ for compact $K\subset A$ and the outer capacity $\mathrm{cap}_e\,(A)$ is equal to $\inf \mathrm{cap}_i\,(\delta)$ for open $\delta\supset A$. Hence, for every open set δ, we have $\mathrm{cap}_i\,(\delta)=\mathrm{cap}_e\,(\delta)$. We shall designate the common value of these two capacities by $\mathrm{cap}\,(\delta)$. We say that a property holds quasi everywhere in Ω if it holds at each point of Ω except at the points of a set of outer capacity zero.

Definition 1. We shall say that a potential U^μ is *quasi continuous* in Ω, if, for any positive number ε, there is an open set δ_ε such that $\mathrm{cap}\,(\delta_\varepsilon)\leqq\varepsilon$ and the restriction of U^μ to $\Omega-\delta_\varepsilon$ is continuous.

Definition 2. We say that Φ satisfies the *quasi continuity principle*, if the continuity of the restriction of any potential U^μ to S_μ implies the quasi continuity of U^μ in Ω.