75. Fourier Series. XVII. Order of Partial Sums and Convergence Theorem

By Masako SATÔ

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1. **Introduction.** Let $f(t)$ be an integrable function with period 2π and its Fourier series be

$$(1)\qquad \frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt).$$

By $s_n(x)$ we denote the nth partial sum of the Fourier series (1). We put as usual $\varphi_x(t)=f(x+t)+f(x-t)$.

We have proved the following theorems in [1].

Theorem 1. *If*

$$(2)\qquad \int_0^t \varphi_x(u)\,du=o(t)\qquad (t\to 0)$$

and

$$(3)\qquad \int_0^t (f(\xi+u)-f(\xi-u))\,du=o(t)\qquad (t\to 0)$$

uniformly in ξ in a neighbourhood of x, then

$$s_n(x)=o(\log n).$$

Theorem 2. *If*

$$(4)\qquad \int_0^t \varphi_x(u)\,du=o\left(t\Big/\log\frac{1}{t}\right)\qquad (t\to 0)$$

and

$$(5)\qquad \int_0^t (f(\xi+u)-f(\xi-u))\,du=o\left(t\log\log\frac{1}{t}\Big/\log\frac{1}{t}\right)\qquad (t\to 0)$$

uniformly in ξ in a neighbourhood of x, then

$$s_n(x)=o(\log\log n).$$

By the same way as the proof of these theorems, we get the following generalizations.

Theorem 3. *Let $0\leqq\alpha\leqq 1$. If*

$$(6)\qquad \int_0^t \varphi_x(u)\,du=o\left(t\left(\log\frac{1}{t}\right)^{\alpha}\right)\qquad (t\to 0)$$

and

$$(7)\qquad \int_0^t (f(\xi+u)-f(\xi-u))\,du=o\left(t\Big/\left(\log\frac{1}{t}\right)^{1-\alpha}\right)\qquad (t\to 0)$$

uniformly in ξ in a neighbourhood of x, then

$$(8)\qquad s_n(x)=o((\log n)^{\alpha}).$$

Theorem 4. *Let $0\leqq\alpha\leqq 1$. If*