72. Fourier Series. XVI. The Gibbs Phenomenon of Partial Sums and Cesàro Means of Fourier Series. 1

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1. There are many literatures concerning the Gibbs phenomenon of partial sums and Cesàro means of Fourier series of functions at jump points but a few concerning that at the points of discontinuity of the second kind (see B. Kuttner [1-4], O. Szász [5], S. Izumi and M. Satô [6] and K. Ishiguro [7, 8]). In our paper [6] we have proved

Theorem 1. *Suppose that*
$$f(t)=a\psi(t-\xi)+g(t)$$
where $\psi(t)$ is a periodic function with period 2π such that
$$\psi(t)=(\pi-t)/2 \qquad (0<t<2\pi)$$
and
$$-a\pi/2\leqq \liminf_{t\to\xi} f(t)\leqq \limsup_{t\to\xi} f(t)\leqq a\pi/2.$$
If
$$\int_0^t g(\xi+u)du=o(|t|),$$
and
$$\int_0^t (g(x+u)-g(x-u))du=o(|t|)$$
uniformly for all x in a neighbourhood of ξ, then the Gibbs phenomenon of $f(t)$ appears at $t=\xi$, and the Gibbs set contains the interval $[-a(H+1)\pi/4,\ a(H+1)\pi/4]$.

Theorem 2. *There is a function which does not present the Gibbs phenomenon at $t=\xi$ and has $t=\xi$ as the second kind discontinuity.*

We shall here prove

Theorem 3. *If*
$$(1)\qquad \int_0^h (f(x+u)-f(x-u))du=o\Big(h\Big/\log\frac{1}{h}\Big),\qquad \textit{uniformly in } x,$$
then the partial sums of Fourier series of $f(t)$ do not present the Gibbs phenomenon at all points.

Using Theorem 3, we give a simple proof of Theorem 2. Further, as a particular case, we get the following theorem.

Theorem 4. *If $f(t)$ is continuous at a point x (or in an interval (α,β) or in $(0,2\pi)$), and (1) holds, then the Fourier series of $f(t)$ converges uniformly at x (or in a closed interval contained in (α,β) or in $(0,2\pi)$).*