71. *On Conformal Mapping of Polygonal Domains*

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1. In a recent paper an explicit formula has been established for an analytic function mapping an annulus onto a domain which contains the point at infinity in its interior and whose boundary consists of two rectilinear polygons (see Satz 8 in [2]). Since the formula has been obtained there as a corollary of a general representation theorem for analytic functions in which the Villat-Stieltjes formula has been taken into account as an essential tool for its proof, it has been pre-announced that another more direct way of proof will be published later. In fact, it is possible to give an alternative proof without any reference to the Villat-Stieltjes representation formula.

Main purpose of the present paper is to fulfil the promise mentioned above. The present method is of primitive nature and is really an analogue of a classical proof in establishing the Schwarz-Christoffel formula for a function mapping a circle onto the exterior of a polygon. Moreover, a related formula for a function mapping an annulus onto a rectilinear polygonal ring domain which does not contain the point at infinity has been already derived by a method which remains valid in the present case with less modification (cf. the proof of Theorem 4 in [1]).

On the other hand, the paper [1] contains further results on mappings onto multiply connected circular polygonal domains. Supplementary remarks will be made below also to some of them.

2. Now an alternative proof will be given below for the theorem in question (Satz 8 in [2]) which may be re-stated as follows:

THEOREM 1. *Let $f(z)$ be an analytic function which maps an annulus $(0<)\, q<|z|<1$ onto a rectilinear polygonal ring domain containing the point at infinity in its interior. Let the vertices of the boundary polygons be designated by $f(e^{i\varphi_\mu})$ $(\mu=1,\cdots,m)$ and $f(qe^{i\psi_\nu})$ $(\nu=1,\cdots,n)$ and the interior angles (with respect to the image-domain) at these vertices by $\gamma_\mu\pi$ and $\delta_\nu\pi$, respectively. Further, let z_∞ be the antecedent of the point at infinity. Then there holds a formula*

$$f(z)=C\int^z z^{ic^*-1}\frac{\prod_{\mu=1}^{m}\sigma(i\lg z+\varphi_\mu)^{\gamma_\mu-1}\prod_{\nu=1}^{n}\sigma_3(i\lg z+\psi_\nu)^{\delta_\nu-1}}{\sigma(i\lg(z/z_\infty))^2\sigma(i\lg(\bar{z}_\infty z))^2}dz+C'$$

with certain constants C and C'. Here the sigma-functions are those