96. Fourier Series. XVIII. On a Sequence of Fourier Coefficients

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1. Let $f(t)$ be an integrable function with period 2π and its Fourier series be

$$\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)=\sum_{n=0}^{\infty}A_n(x).$$

Then the derived series is

$$\sum_{n=1}^{\infty}(b_n\cos nx-a_n\sin nx)=\sum_{n=1}^{\infty}B_n(x).$$

L. Fejér [1] (cf. [6, p. 62]) proved that, if $l=f(x+0)-f(x-0)$ exists and is finite, then the sequence $\{nB_n(x)\}$ converges (C,r) $(r>1)$ to l/π. Later many writers treated the Cesàro convergence of the sequence $\{nB_n(x)\}$. Recently B. Singh [2] has proved the following theorem.*)

Theorem. *If*

$$\int_0^t\psi_x(u)\,du=o(t),\qquad \psi_x(t)=f(x+t)-f(x-t)-l,$$

and

$$\lim_{\varepsilon\downarrow 0}\int_\varepsilon^\delta\frac{|\psi_x(t+\varepsilon)-\psi_x(t)|}{t}dt=0,$$

where δ is a fixed positive number, then the sequence $\{nB_n(x)\}$ converges $(C,1)$ to the value l/π.

We shall prove the following theorems.

Theorem 1. *Let $0\leqq\alpha\leqq 1$. If*

$$\Psi_x(t)=\int_0^t\psi_x(u)\,du=o\left(t\left(\log\frac{1}{t}\right)^\alpha\right)$$

and

$$\int_0^t(\psi_x(\xi+u)-\psi_x(\xi-u))\,du=o\left(t\Big/\left(\log\frac{1}{t}\right)^{1-\alpha}\right)$$

uniformly in ξ, then $\sigma_n(x)-l/\pi=o((\log n)^\alpha)$ where $\sigma_n(x)$ is the nth $(C,1)$ mean of $\{nB_n(x)\}$.

Theorem 2. *Let $0\leqq\alpha\leqq 1$. If*

$$\Psi_x(t)=o\left(t\left(\log\log\frac{1}{t}\right)^\alpha\right)$$

and

$$\int_0^t(\psi_x(\xi+u)-\psi_x(\xi-u))du=o\left(t\left(\log\log\frac{1}{t}\right)^\alpha\Big/\log\frac{1}{t}\right)$$

uniformly in ξ, then $\sigma_n(x)-l/\pi=o((\log\log n)^\alpha)$.

*) Concerning the earlier references, see [2-4].