92. *AU-covering Theorem and Compactness*

By Kiyoshi ISÉKI
Kobe University

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To characterize a countably compact normal space, the present author introduced the concept of *AU*-property of covering [2-7]. In this paper, we shall discuss the *AU*-covering theorem of a topological space.

Let S be a topological space, and let Φ be an open covering of S. By $\overline{\overline{\Phi}}$, we denote the power of the set of elements of Φ. A covering Φ is said to be *AU-reducible*, if it contains a subfamily Ψ of lower power than $\overline{\overline{\Phi}}$ such that the union of the closures of elements of Ψ is S, otherwise it is *AU-irreducible*.[1]

For our discussion, we shall use some classical concepts which were introduced by E. W. Chittenden [1].

A covering Φ with power $\aleph_\mu$ is normal, if it can be well-ordered in the form

$$O_1, O_2, \cdots, O_\alpha, \cdots \qquad (\alpha<\omega_\mu)$$

where ω_μ is the initial ordinal of $\aleph_\mu$ and, for each α, there is an element p_α such that $p_\alpha \in \overline{O}_\alpha$ and $p_\alpha \bar{\in} \overline{O}_\beta$ $(\beta<\alpha)$. The set $\{p_\alpha \mid \alpha<\omega_\mu\}$ is said to be an *associate set* of the normal covering Φ. It is clear that *every normal covering is AU-irreducible*. Now we shall prove the following fundamental

Theorem 1. Every open covering Φ contains a normal covering.

Proof. Let Φ_1 be an *AU*-irreducible subcovering of Φ, and let

$$(1) \qquad O_1, O_2, \cdots, O_\mu, \cdots \qquad (\alpha<\omega_\mu)$$

be a well-ordered set of the type ω_μ of Φ_1. To construct a normal subfamily Ψ of Φ_1, let $O_{\alpha_1}=O_1$, and take $p_2 \in S-\overline{O}_{\alpha_1}$, let O_{α_2} be the first term of the transfinite sequence (1) such that $O_\alpha \ni p_2$. Suppose that O_{α_ν} $(\nu<\beta)$ is defined, then we shall define O_{α_β} as follows: take $p_\beta \in S-\bigcup_{\nu<\beta} \overline{O}_{\alpha_\nu}$, let O_{α_β} be the first term of (1) such that $\overline{O}_\alpha \ni p_\beta$. Since Φ_1 is irreducible, α_β $(\beta<\omega_\mu)$ is defined and $\{\alpha_\beta\}$ is cofinal with $\{\alpha \mid \alpha<\omega_\mu\}$. Therefore $\Psi=\{O_{\alpha_\beta} \mid \beta<\omega_\mu\}$ is a normal covering of power $\aleph_\mu$. Q.E.D.

If an open covering Φ is locally finite (or star finite), then $\overline{\Phi}$ consisting of the closures of all elements of Φ is locally finite (or star finite) and the union of the closures of some elements of Φ is closed.

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1) For the usual concept of reducibility, see E. W. Chittenden [1].