91. Notes on Knots and Periodic Transformations

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Introduction. Let T be a sense preserving periodic transformation of the 3-sphere S^3 onto itself. Furthermore let T be different from the identity and have at least one fixed point. Then it has been shown by Smith[9] that the set F of all fixed points of T is a simple closed curve. Recently Montgomery, Zippin and Samelson[5][6] have studied about the position of F in S^3, which also be concerned in this note. Hereafter we always assume that T is semilinear, and then F is polygonal. Let p be the period of T. Identifying the points

$$x,\ T(x), \cdots, T^{p-1}(x)$$

in S^3, we have an orientable 3-manifold M. Then it will be proved in § 4 that M is simply connected, i.e. the fundamental group of M consists of only one element. In § 5, under the assumption that the well-known Poincaré conjecture on 3-manifolds is true, we shall prove that almost all knots of the Alexander-Briggs's table[1] are not equivalent to F, if T is of period 2. This will be done by the use of Alexander polynomials.[2] To prove these we shall study knots in 3-manifolds in §§ 1–3. *In this note everything will be considered from the semilinear point of view.*

§ 1. Let M be a compact 3-manifold (without boundary) and k an oriented simple closed curve in M. The fundamental group of $M-k$ will be denoted by $F(M-k)$ or sometimes by $F(k, M)$. *Hereafter we always assume that k is homologous to* 0 *in M.* Let V be a sufficiently small tubular neighbourhood of k in M. Then the boundary $\dot{V}$ of V is a torus. A meridian of $\dot{V}$ is a simple closed curve on $\dot{V}$ which bounds a 2-cell in V but not on $\dot{V}$. Let x be an oriented meridian of $\dot{V}$. Since k is homologous to 0 in M, the linking number $\mathrm{Link}(k, x)$ of k and x can be defined and is equal to ± 1. We may always suppose that x is oriented such that

$$\mathrm{Link}(k, x) = 1.$$

For each integer $p(\neq 0)$ x^p is not homotopic to 1.

Now let $\{x, X_1, X_2, \cdots, X_n\}$ be the set of generators of $F(M-k)$. Put

$$\mathrm{Link}(k, X_i) = v(i) \qquad (i=1, 2, \cdots, n)$$

and

$$x_i = x^{-v(i)} X_i. \qquad (i=1, 2, \cdots, n)$$