90. *On AU-property and Countably Compactness*

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It is the purpose of this Note to point out that an example due to A. Ramanthan [2] *is not countably compact semi-regular space with the AU-property* (for the definition, see K. Iséki [1]). Some writers*) have shown that there exist pseudo-compact spaces which are not countably compact. Therefore there exist regular spaces with the AU-property for countable open covering, but not countably compact.

Let a countable space S be defined as follows:

$$S=\{a\}\smile\{b\}\smile\{a_{ij}\}\smile\{b_{ij}\}\smile\{c_i\}. \quad (i, j=1, 2, \cdots)$$

The neighbourhoods of each point of S are defined:

$$U(a_{ij})=\{a_{ij}\}, \quad U(b_{ij})=\{b_{ij}\},$$

$$U_n(c_i)=\{c_i\}\smile\bigcup_{j=n}^{\infty}(\{a_{ij}\}\smile\{b_{ij}\}),$$

$$U_n(a)=\{a\}\smile\bigcup_{j=1}^{\infty}\bigcup_{i=n}^{\infty}a_{ij},$$

$$U_n(b)=\{b\}\smile\bigcup_{j=1}^{\infty}\bigcup_{i=n}^{\infty}b_{ij},$$

where $n=1, 2, \cdots$. It is easily seen that every countable open covering has the AU-property, and $\bigcup_{i=1}^{\infty}c_i$ is a countable set without cluster point. Therefore, S is the required properties.

References

[1] K. Iséki: A remark on countably compact normal space, Proc. Japan Acad., **33**, 131–133 (1957).
[2] A. Ramanthan: Maximal Hausdorff spaces, Proc. Indian Acad. Sci., **26**, 31–42 (1947).

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*) The examples of such spaces have been given by E. Hewitt, J. Novák, S. Mrówka, V. Ptak, W. T. van Est and H. Freudenthal.