# 114. *On B-covers and the Notion of Independence in Lattices*

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Introduction. In [3], L. M. Kelley has introduced the concept of $B$-covers as metric-between in a normed lattice. We have extended this notion to the case of general lattices in [4] and studied the geometries in lattices by means of $B$-covers and $B^*$-covers in [5]. In the first section of this paper we shall show that the relation "relative modularity" or "relative independence" which is derived from Wilcox [1] has a close connection with the $J$-cover or the $CJ$-cover which is a part of the $B$-cover in lattices. In the second section we shall consider the relations between the $B$-covers and independent sets in lattices.

For any two elements $a, b$ of a lattice $L$, we shall define as follows.

$J(a,b)=\{x\,|\,(a\frown x)\smile(b\frown x)=x,\ x\in L\}$, $CJ(a,b)=\{x\,|\,(a\smile x)\frown(b\smile x)=x,\ x\in L\}$. $J(a,b)$ is called the $J$-cover of $a$ and $b$, and if $x\in J(a,b)$, then we shall write $J(axb)$. Similarly we shall define $CJ$-cover and $CJ(axb)$.

$B(a,b)=J(a,b)\frown CJ(a,b)$ is called the $B$-cover of $a$ and $b$ and we shall write $axb$ when $x\in B(a,b)$ (cf. [4, 5]).

1. Relative modular pairs and $J$-covers ($CJ$-covers). Following L. R. Wilcox [1], $(a,b)$ is called a modular pair when $x\leqq b$ implies $(x\smile a)\frown b=x\smile(a\frown b)$, and in this case we write $(a,b)M$. In [5] we defined a relative modular pair $(a,b)M^*$ to be a pair $(a,b)$ such that $a\frown b\leqq x\leqq b$ implies $(x\smile a)\frown b=x\smile(a\frown b)$.

$B$-covers treat "between" in lattices (cf. [4, 5]), while $J$-covers and $CJ$-covers may be considered as describing "semi-between" in lattices.

In the following $L$ is always assumed to be a lattice.

Lemma 1.1. *The following statements are equivalent in case* $b'\leqq b$:

(a) $(b'\smile a)\frown b=b'\smile(a\frown b)=b$. $((b'\smile a)\frown b=b'\smile(a\frown b)=b')$.

(b) $J(abb')$ $(CJ(ab'b))$.

Proof. If $(b'\smile a)\frown b=b'\smile(a\frown b)=b$, then we have $(a\frown b)\smile(b\frown b')=(a\frown b)\smile b'=b$, that is $J(abb')$. Conversely if $J(abb')$, then we have $b=(a\frown b)\smile(b\frown b')\leqq b\frown(a\smile b')\leqq b$, and hence we have $(b'\smile a)\frown b=b=b'\smile(a\frown b)$. Similarly we can treat the other case.

Theorem 1.1. *If* $J(abb')$ (*resp.* $CJ(ab'b)$) *holds for any* $b'$ *with* $b'\leqq b$ *then we have* $(a,b)M$.