113. *The Initial Value Problem for Linear Partial Differential Equations with Variable Coefficients. III*

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In the present paper we consider mixed problems of linear parabolic equations with boundary conditions formulated by J. L. Lions [4] such that, using his notations, V is independent of the time variables, but N_t depends on them.

Our methods (§2), are also applicable to mixed problem of linear equations of many other types with above-mentioned boundary conditions, with which it seems interesting to me to compare Kato's methods [2].

As an illustlation of our considerations we consider in §3 the Fokker-Planck's equations formulated by K. Yosida [7].

Only a sketch of this proof will be given, however, the details with further investigations will be published elsewhere.

1. Preliminary. Let Ω be a domain of the Euclidean space. Let $((u, v))_t$ be real bilinear forms defined on a real separable Hilbert space V with following conditions, where $\mathfrak{D}(\Omega)\subset V\subset L_2(\Omega)$ and the injections $\mathfrak{D}(\Omega)\to V$, $V\to L_2(\Omega)$ are both continuous: there are positive constants a, b, c such that for any $t(-\infty<t<\infty)$

(I)
$$((u, u))_t \geqq a\,\|u\|_V^2$$
$$b\,\|u\|_V\,\|v\|_V \geqq |((u, v))_t|,$$

(II) for fixed $u, v\in V$,
$$c\,|t-t'|\,\|u\|_V\,\|v\|_V \geqq |((u, v))_t-((u, v))_{t'}|.$$

Furthermore let $\bar{A}_t$ be an operator in $L_2(\Omega)$ into itself such that $f\in D(\bar{A}_t)$ if and only if $((f, v))_t=(\bar{A}_t f, v)_{L_2(\Omega)}$ for every $v\in V$, where $(\bar{A}_t f, v)_{L_2(\Omega)}=A_t f(v)$ for the distribution $A_t f$ defined by the relation: $((f, v))_t=A_t f(v)$ for every $v\in\mathfrak{D}(\Omega)$. Then $\bar{A}_t$ is a densely defined, closed operator in $L_2(\Omega)$ into itself whose adjoint coincides with operator $\bar{A}_t^*$ defined as above from $((u, v))_t^*=((v, u))_t$ [3, 4]. Let G_t^* be the Green operators with respect to the form $((u, v))_t^*$. Then from (II) we see the following

Lemma 1. *For any $u\in L_2(\Omega)$, $G_t^* u$ is differentiable in V (a.e.t) and $\frac{d}{dt}G_t u$ is measurable with respect to V such that*

$$\left\|\frac{d}{dt}G_t u\right\|_V \leqq c\,\|u\|_V \qquad (a.e.t).$$

Definition. Let E be a real separable Hilbert space. Then we denote by $\mathfrak{L}^n(E)$ the completion of the real linear space $\mathfrak{D}_t(E)$ with