111. A Note on Some Topological Spaces

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This short note has two purposes: one of them is to determine a topological space treated in [2], that is, it will be shown below that a Hausdorff space satisfying one of the conditions listed in Theorem 2 of [2] is nothing more than a finite set; and the other is to point out a more fundamental property of weakly compact spaces.*)

Let $\mathfrak{S}=\{O_\alpha\}_{\alpha\in A}$ be a family of subsets of a topological space E. We say that $\mathfrak{S}$ is *point finite* if each point of E belongs at most to a finite number of the members of $\mathfrak{S}$. The family $\mathfrak{S}$ is said to be *locally finite* if each point of E possesses a neighbourhood which intersects at most finitely many members of $\mathfrak{S}$; and $\mathfrak{S}$ is *star finite* if each member of $\mathfrak{S}$ intersects at most finitely many members of $\mathfrak{S}$. Moreover, $\mathfrak{S}$ is termed *weakly locally finite* if $\mathfrak{S}$ is locally finite as a family of subsets of the subspace $\bigcup_{\alpha\in A} O_\alpha$ of E (i.e. if each point of $\bigcup_{\alpha\in A} O_\alpha$ possesses a neighbourhood which intersects finitely many members of $\mathfrak{S}$). If the set A of indices is a finite set, the family $\mathfrak{S}$ is called *finite*. Obviously, a star finite family is weakly locally finite, a locally finite family is weakly locally finite, and a weakly locally finite family is point finite.

THEOREM 1. *The following conditions on a Hausdorff space E are equivalent*:

(1) *Every point finite open covering of E is finite.*

(2) *Every point finite family of open sets of E is finite.*

(3) *Every weakly locally finite family of open sets of E is finite.*

(4) *Every star finite family of open sets of E is finite.*

(5) *Every family of pairwise disjoint open sets of E is finite.*

(6) *E is a finite set.*

Proof. It will suffice to prove that (5) implies (6). To prove this, it is sufficient to show that each point of E is open. Suppose that there exists a point $x\in E$ which is not open. Then, if V_0 is a neighbourhood of x, we can find a point $x_1\in V_0$ distinct from x, and then we can choose disjoint open sets V_1 and O_1 such that $x\in V_1$, $x\in O_1$ and $V_1\subseteq V_0$, $O_1\subseteq V_0$. Thus, by induction, it is easy to construct a sequence $\{V_n\}$ of neighbourhoods of x and a sequence $\{O_n\}$ of open

*) For the definition of weakly compact space (espace faiblement compact), see [3 or 4].