128. On Non-linear Partial Differential Equations of Parabolic Types. I

By Haruo MURAKAMI
Kobe University

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Introduction. In this paper we shall consider the following non-linear partial differential equations of parabolic types:

$$\partial^2 u/\partial x^2 - \partial u/\partial y = f(x, y, u)$$
$$\partial^2 u/\partial x^2 - \partial u/\partial y = f(x, y, u, \partial_x u),$$
$$\partial^2 u/\partial x^2 - \partial u/\partial y = f(x, y, u, \partial_x u, \partial_y u).[1]$$

Our main aim is to solve the first boundary value problem of the first equation by so-called Perron's method which was originally used by O. Perron to solve the Dirichlet problem for Laplace's equation[2] and later used by W. Sternberg for the equation of heat conduction.[3] Recently, Prof. T. Satō modified this method and solved the Dirichlet problem for the non-linear equation of elliptic type.[4] In his papers, however, as an inevitable consequence of the method used there and of the non-linearity of the equation, he had to extend the meaning of the Laplacian operator. To solve our problem following Satō's idea, we must also extend the parabolic differential operator $\partial^2/\partial x^2 - \partial/\partial y$ to a generalized heat operator $\square$. This generalization is shown in §1. Thus, the equations considered in this paper are of the following types:

$$(\mathrm{E}_1) \qquad \square u = f(x, y, u),$$
$$(\mathrm{E}_2) \qquad \square u = f(x, y, u, \partial_x u),$$
$$(\mathrm{E}_3) \qquad \square u = f(x, y, u, \partial_x u, \partial_y u).$$

In §1, after the definition of generalized heat operator $\square$, we give some notations and definitions needed in the sequel. In §2 we state and prove some comparison theorems. Theorem 2.7 and its corollaries play important roles later. In §3 we give a uniqueness condition. In §4 we give some existence theorems which show the existence of solutions under some restricted conditions.[5] Harnack's first

1) We use the notations $\partial_x u$ and $\partial_y u$ for $\partial u/\partial x$ and $\partial u/\partial y$ respectively.

2) O. Perron: Eine neue Behandlung der ersten Randwertaufgaben für $\triangle u = 0$, Math. Zeitschr., **18** (1923).

3) W. Sternberg: Ueber die Gleichung der Wärmeleitung, Math. Ann., **101** (1929).

4) T. Satō: Sur l'équations aux dérivées partielles $\triangle z = f(x, y, z, p, q)$, Comp. Math., **12** (1954) and Sur l'équation aux dérivées partielles $\triangle z = f(x, y, z, p, q)$ II (to appear). See also M. Hukuhara and T. Satō: Theory of Differential Equations (in Japanese), Kyōritu Publ. Co. Ltd., Tokyo (1957), cited as Hukuhara-Satō.

5) In his paper which was sent to Prof. T. Satō recently, Prof. B. Pini also proves similar theorems in §§2, 3 and 4 of the present paper independently. Therefore we shall omit the details of the proofs there. See B. Pini: Sul primo problema di valori al contorno per l'equazione parabolica non lineare del secondo ordine, Rend. del Sem. Mat. d. Università di Padova (1957), cited as B. Pini [1].