126. On Generalized Continuous Convergence

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Recently, H. Schaefer [4] introduced a notion of generalized continuous convergence for a sequence of functions on a topological space. On the other hand, the present writer [1] proved that pseudo-compact completely regular space was characterised by theorems of Ascoli type and Arzela type. In this Note, we shall show that such a space is characterised by a notion of *generalized continuous convergence* (in the sense of H. Schaefer) which is defined below. Let S be a T_2-space, and let $f(x)$ be a real function defined on S. For a point x_0 of S, we shall define the *oscillation* $\sigma(f, x_0)$ of $f(x)$ at the point x_0 as follows:

$$\sigma(f, x_0)=\inf_{U} \sup_{x,x'\in U} |f(x)-f(x')|,$$

where inf. takes all neighbourhood U of x_0.

Let U be a neighbourhood of x_0, and $f_n(x)$ be a sequence of functions defined on S, then we set $S(n, U)=\bigcup_{k\geq n} f_k(U)$. The set $\{S(n, U)\}$ for n and U is *a base of filter on real line.* We denote the filter generated by $\{S(n, U)\}$ by $F(x_0, f_n)$.

Definition 1. *A sequence* $\{f_n(x)\}$ *of functions is said to be continuously convergent at a point* x_0 *of* S, *if the filter* $F(x_0, f_n)$ *converges on reals* (see H. Schaefer [4, p. 424]).

$f_n(x)$ is said to be *uniformly convergent at a point* $x_0\in S$, if for a given positive ε, there are a neighbourhood U of x_0 and an integer N such that $|f_m(x)-f_n(x)|<\varepsilon$ for all x of U and $m, n\geq N$.

In his paper [1], the present author has proved that *a completely regular space is pseudo-compact if and only if every sequence of continuous functions which is uniformly convergent at each point is uniformly convergent.*

H. Schaefer [4] has obtained that a sequence $\{f_n(x)\}$ of continuous functions on a topological space converges continuously at a point x_0 if and only if it is uniformly convergent at the point x_0 and $\lim_{n\to\infty}(x_0, f_n)=0$.

Therefore, if a completely regular space S is pseudo-compact, and a sequence $\{f_n(x)\}$ of continuous functions is continuously convergent at each point, its convergence is uniform on S. Conversely, if every sequence of continuous functions which converges continuously at each point on a completely regular space is uniformly convergent, then it is pseudo-compact. Suppose that it is not pseudo-compact, then there