124. Non-Connection Methods for the Theory of Principal Fibre Bundles as Almost Kleinean Geometries

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In some of the previous papers of the present author (T. Takasu [8, 9]), the author has established, on one hand, six non-holonomic geometries as double geometries consisting of the respective connection geometries and the respective non-holonomic geometries referred to the connection parameters of the teleparallelisms (É. Cartan [1]), having discovered the most remarkable fact that *the paths of the teleparallelisms* (i.e. the II-geodesic curves) *behave as for meet and join like straight lines*, what has led the author to the discovery of the actual and final formulation of the general theory of relativity as the 3-dimensional Laguerre principal fibre bundle geometry (T. Takasu [11]).

S. S. Chern and C. Ehresmann (S. S. Chern [2]; C. Ehresmann [3, 4]; A. Lichnerowicz [5]; K. Nomizu [6]; T. Ohtsuki [7]) established, on the other hand, a theory of connections as that of the cross sections of the (principal) fibre bundles introducing connections into them.

In this note, it will be shown firstly that *the present author's theory of the respective principal fibre bundles (based on the II-geodesic curves) is substantially nothing other than the respective theory in the large of S. S. Chern or that of C. Ehresmann*, since the II-geodesic curves do actually exist in the differentiable manifolds in the sense of them. Indeed É. Cartan [1] has once declared: "Les connexions affines que j'ai introduites rentrent dans les connexions encore plus générales dues à M. Schouten (Math. Zeitschr., 13, 56–81 (1922)); mais le point de vue de M. Schouten est différent du mien. Pour lui le transport parallèle (lineare Übertragung) est la notion géométrique essentielle; *pour moi, elle n'est qu'un moyen qui tient aux propriétés de l'espace affine et qui ne peut plus s'utiliser, au moins directement, pour établir la notion d'espace à connexion projective (ou conforme, etc.)*" and *the present author has the same notion as É. Cartan had, for, the choice of a connection for one and the same differentiable manifold corresponding to a Lie group means a choice of the paths as tangents to given curves and given subvarieties.*

It will also be shown secondly that the present author's theory of the respective principal fibre bundles based on the II-geodesic curves provides us *non-connection methods for the differentiable manifolds admitting infinitely many connections* and that *the results reduce to such an extent that the geometries under consideration become the cor-*