147. *On Non-linear Partial Differential Equations of Parabolic Types. II*

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As stated in the Introduction of the previous paper,[1] we give here some uniqueness conditions, existence theorems (I) and some preparatory theorems for the main existence theorem which will be given in the next part.

3. Uniqueness conditions. LEMMA. *Let $f(x,y,u,p)$ be defined on $(x,y)\in(C,S]$, $-\infty<u,p<+\infty$. If*

$$f(x,y,u,p)\begin{cases} >0 & u>0 \\ =0 & u=0 \\ <0 & u<0, \end{cases} \tag{3.1}$$

then there is one and only one solution of (E_2) which is continuous on $[C,S]$ and which vanishes on C.

DEFINITION. *Let $f(x,y,u,p)$ be a function defined on $(x,y)\in(C,S]$, $-\infty<u$, $p<+\infty$. We say that $f(x,y,u,p)$ satisfies the condition (Lk) if there exists a positive constant k such that*

$$f(x,y,u_1,p)-f(x,y,u_2,p)>-k(u_1-u_2) \tag{Lk}$$

for $(x,y)\in(C,S]$ and $u_1>u_2$.

REMARK. If we set $v=ue^{-ky}$, by simple calculation, we have

$$\begin{aligned} \overline{\Box}v(x,y)&\le ke^{-ky}u(x,y)+e^{-ky}\overline{\Box}u(x,y), \\ \underline{\Box}v(x,y)&\ge ke^{-ky}u(x,y)+e^{-ky}\underline{\Box}u(x,y). \end{aligned} \tag{3.2}$$

Then the equation (E_2) is written by

$$\Box v=F(x,y,v,\partial_x v) \tag{3.3}$$

where

$$F(x,y,v,p)=kv+e^{-ky}f(x,y,ve^{ky},pe^{ky}). \tag{3.4}$$

If we assume that $f(x,y,u,p)$ satisfies the condition (Lk)

$$\begin{aligned} &F(x,y,v_1,p)-F(x,y,v_2,p) \\ &=k(v_1-v_2)-e^{-ky}\{f(x,y,v_1e^{ky},pe^{ky})-f(x,y,v_2e^{ky},pe^{ky})\} \\ &>k(v_1-v_2)-k(v_1-v_2)=0 \end{aligned}$$

for $v_1>v_2$, so that $F(x,y,v,p)$ is *monotone increasing* (*strictly*) *with respect to v.*

B. Pini proved in his paper[2] that (E_2) has at most one solution which is continuous on $[C,S]$ and which admits the prescribed continuous boundary value if $f(x,y,u,p)$ is monotone increasing with

1) Proc. Japan Acad., **33**, 530–535 (1957).

2) B. Pini: Sul primo problema di valori al contorno per l'equazione parabolica non lineare del secondo ordine, Rend. del Sem. Mat. Università di Padova, 153 (1957).