140. On Eigenfunction Expansions of Self-adjoint Ordinary Differential Operators. I

By Takashi KASUGA

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In this note, we shall prove some results about eigenfunction expansions of self-adjoint ordinary differential operators for the case when one of their characteristic functions[1] is meromorphic on some parts of the real line R.

§1. Let us consider the differential expression

$L[u]=-(d/dx)\{p(x)d/dx\}u+q(x)\cdot u \quad (a<x<b, -\infty\leqq a<b\leqq+\infty)$

defined on a (finite or infinite) open interval (a,b), where $p(x)$, $q(x)$ are real-valued functions defined in (a,b), $p(x)$ has continuous first derivative, $q(x)$ is continuous, and $p(x)>0$ for $a<x<b$.

Following H. Weyl,[2] we classify L according to its behaviour in the neighbourhood of the point a (or b), in *the l. c. type* (*limit circle type*) at a (or b) and *the l. p. type* (*limit point type*) at a (or b).

In this note, all functions are complex-valued if not specially noted.

$\mathfrak{H}_I$: the set of functions defined on (a,b) and square summable on I, where I is a subinterval (open, closed, or half-open) of (a,b).

$\mathfrak{H}$: the set $\mathfrak{H}_{(a,b)}$ of functions.

$\mathfrak{D}$: the set of functions u defined on (a,b) such that u is differentiable on (a,b) and du/dx is absolutely continuous on every finite closed subinterval of (a,b).

$\mathfrak{G}_a$ (or $\mathfrak{G}_b$): the set of functions belonging to $\mathfrak{D}$ such that u, $L[u]\in\mathfrak{H}_{(a,c]}$ (or $\mathfrak{H}_{[c,b)}$) for every point c of (a,b).

Bracket. For $u,v\in\mathfrak{D}$, we introduce the bracket:

$$[uv](x)=p(x)[u(x)v'(x)-v(x)u'(x)]$$
$$(u'=du/dx,\ v'=dv/dx).$$

In case u and v satisfy one and the same equation $L[u]=l\cdot u$, we write $[uv]$ for $[uv](x)$, since, in this case $[uv](x)$ does not depend on x.[3]

If L is of the l. c. type at a (or b) and $u,w\in\mathfrak{G}_a$ (or $\mathfrak{G}_b$), the limit $[wu](a)=\lim_{x\to a}[wu](x)$ (or $[wu](b)=\lim_{x\to b}[wu](x)$) exists.[4]

Boundary conditions.

$\mathfrak{G}'_a$ (or $\mathfrak{G}'_b$): the set $\mathfrak{G}_a$ (or $\mathfrak{G}_b$) of functions if $L[u]$ is of the l. p.

1) Cf. §1.
2) Cf. Weyl [7], Titchmarsh [6], Coddington and Levinson [2].
3) Cf. the reference quoted in 2).
4) Cf. the reference quoted in 2).