4. A Note on the Integration by the Method of Ranked Spaces

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§1. Prof. K. Kunugi showed in his note "Application de la méthode des espaces rangés à la théorie de l'intégration. I"[1] that a new integration can be constructed by the method of ranked spaces,[2] and suggested that the development of his theory could be generalized for functions on abstract spaces—for example, locally compact topological groups. In this note, we shall consider the locally compact group G and we shall show that the construction of integrals can be done without changing any detail of the preceding note.

Let G be a locally compact group, m be a Haar measure in G,[3] that is, a Borel measure in G, such that $m(U)>0$ for every non empty Borel open set U, and $m(xE)=m(E)$ for every Borel set E, and for every element x of G.

First we shall remark that, in a locally compact group there is a fundamental system of neighbourhoods of unit element e, which consists of neighbourhoods whose boundaries are of measure zero.

Let V be a compact neighbourhood of unit element e whose boundary is of measure zero, and from now on our considerations are restricted to the fixed V.

Let the family $\mathcal{O}$ be a totality of open sets in V whose boundaries are of measure zero. Then,

(1) If $O_1\in\mathcal{O}$, $O_2\in\mathcal{O}$ then $O_1\smile O_2\in\mathcal{O}$, $O_1\frown O_2\in\mathcal{O}$.

(2) If $O_1\in\mathcal{O}$, $O_2\in\mathcal{O}$ then $O_1\frown(V-\overline{O}_2)\in\mathcal{O}$.

The vector space over the field of real numbers generated by characteristic functions of sets in $\mathcal{O}$ is denoted by Φ. To $f\in\Phi$ correspond a finite number of disjoint sets $O_i\in\mathcal{O}$ $(i=1,2,\cdots,n)$ and

$$f(x)=\sum_{i=1}^{n}\alpha_i\chi_{0_i}(x)$$

where χ_{0_i} is a characteristic function of O_i, and α_i is a real number. Two functions of Φ, $f(x)$, $g(x)$ are identified when they are different only on the boundary of $O\in\mathcal{O}$. Obviously if $f\in\Phi$, $g\in\Phi$ then $f+g\in\Phi$,

1) K. Kunugi: Application de la méthode des espaces rangés à la théorie de l'intégration. I, Proc. Japan Acad., **32**, 215-220 (1956).

2) K. Kunugi: Sur les espaces complets et régulièrement complets. I, II, Proc. Japan Acad., **30**, 553-556, 912-916 (1954).

3) On Haar measure, see for example P. R. Halmos; Measure Theory, New York (1950).