18. *Quasiideals in Semirings without Zero*

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O. Steinfeld [2, 3] has introduced the notion of quasiideals in rings, and semigroups and proved some interesting theorems. In this paper, we shall consider and prove some theorems on quasiideals in semirings. For fundamental concepts on a semiring and its related subjects, we shall follow the papers by S. Bourne [1], H. S. Vandiver and M. W. Weaver [4]. Unless otherwise stated, the word *semiring* shall mean *semiring without zero*.

Let S be a semiring, and suppose that A is a subset of S which is additively closed: if $a, b \in A$, then $a+b \in A$. A is a *quasiideal* if and only if $AS \frown SA \subset A$. Any quasiideal A is subsemiring of S, since $A^2 \subset AS \frown SA \subset A$. The intersection $\bigcap_\alpha A_\alpha$ of quasiideals A_α of S is empty or a quasiideal. For, if $A=\bigcap_\alpha A_\alpha \neq \phi$, then, for each α, $AS \frown SA \subset A_\alpha S \frown SA_\alpha \subset A_\alpha$, and we have $AS \frown SA \subset A$.

Lemma 1. The intersection of a right ideal and a left ideal in a semiring is a quasiideal.

Proof. Let R be a right ideal in S, and L a left ideal in S, then $RL \subset R \frown L$ and $R \frown L$ is not empty. Further, we have

$$(R\frown L)S \frown S(R \frown L) \subseteq RS \frown SL \subseteq R \frown L,$$

and this shows that $R \frown L$ is a quasiideal.

Lemma 2. Let ε be a multiplicative idempotent, and L a left ideal, R a right ideal in a semiring S, then εL and $R\varepsilon$ are quasiideal and

$$\varepsilon L = L \frown \varepsilon S, \quad R\varepsilon = S\varepsilon \frown R.$$

Proof. By Lemma 1, it is sufficient to prove the relations $\varepsilon L = L \frown \varepsilon S$ and $R\varepsilon = S\varepsilon \frown R$. As it is trivial that $\varepsilon L \subseteq L \frown \varepsilon S$, we shall show $\varepsilon L \supseteq L \frown \varepsilon S$. Let a be an element of $L \frown \varepsilon S$, then we have

$$a = \varepsilon S,$$

$s \in S$ and $a \in L$.
Hence, since $\varepsilon^2 = \varepsilon$, we have

$$\varepsilon a = \varepsilon \cdot \varepsilon S = \varepsilon S$$

and this shows $\varepsilon s = \varepsilon a \in \varepsilon L$ and we have $L \frown \varepsilon S \subset \varepsilon L$, similary, for right ideal R, we have $R\varepsilon = S\varepsilon \frown R$.

Theorem 1. The intersection of minimal right and minimal left ideals in a semiring is a minimal quasiideal.

Proof. Let R and L be minimal right and left ideals in the semiring S, and let Q be the intersection of R and L, then Q is a non-