17. On Some Function Spaces concerning Dirichlet's Problem

By Shin-ichi MATSUSHITA

(Comm. by K. KUNUGI, M.J.A., Feb. 12, 1958)

§ 1. Let E be the n-dimensional Euclidean space for a certain n ($\geqq 3$), and denoting the Euclidean distance in E by $r(x, y)$ we define the Newtonian potential

$$\phi(\mu)(x)=N_n\int r^{2-n}(x, y)\,d\mu(y), \quad N_n=\frac{\Gamma(n/2)}{2(n-1)\pi^{n/2}}$$

for any positive Radon measure μ in E.

Let D be a given domain in E, whose closure $\overline{D}$ and hence boundary ∂D are both compact. For each positive measure μ distributed in $\overline{D}$, consider the *inner balayage* μ_Γ^0 of μ in ∂D and the *outer balayage* μ_∇^0 of μ in $\partial\overline{D}$ (about these matters, see my another paper "On the foundation of balayage theory" which will appear in the Journal of Polytech., Osaka City Univ., 9, no. 2; cited hereafter as [1]). The notations and results used here shall be referred to that paper [1], but some of those are quoted for convenience' sake as follows.

For a measurable set X in E, we define

$C(X)$ (or $C_u(X)$)=space of all bounded (resp. uniformly) continuous functions defined in X,

$\mathfrak{M}^+(X)$ (or $\mathfrak{M}_0^+(X)$)=collection of all positive Radon measures distributed in X (resp. of norm less that 1). $\mathfrak{M}(X)$=linear envelope of $\mathfrak{M}^+(X)$ on the reals.

Γ_0=set of all inner regular boundary-points of D (*i.e.* $(\varepsilon_x)_\Gamma^0=\varepsilon_x$ whenever $x\in\Gamma_0$).

∇_0=set of all outer regular (or in other words, *stable*) boundary-points of $\overline{D}$ (*i.e.* $(\varepsilon_x)_\nabla^0=\varepsilon_x$ whenever $x\in\nabla_0$).[1]

$H(D)$=normed linear space consisting of the restrictions in $\overline{D}$ of all bounded potentials $f=\phi(\mu)$ for $\mu\in\mathfrak{M}(E-D)$ with respect to the norm

$$(1.1)\qquad \|f\|_D=\sup_{x\in\overline{D}}|f(x)|.$$

We see that $\nabla_0\subset\Gamma_0\subset\partial D$ and $\partial D-\Gamma_0$ is of (inner) capacity 0.

§ 2. We now define a linear normed space $\Phi(\Gamma_0)$ linearly generated on Γ_0 from the collection of all potentials $\phi_x=\phi(\varepsilon_x)$ for $x\in E-\overline{D}$ with respect to the norm

$$(2.1)\qquad \|\phi_x\|_{\Gamma_0}=\sup_{y\in\Gamma_0}|\phi_x(y)|.$$

1) ε_x designates a point measure of total mass $+1$ placed on $x\in E$.