48. *Measures in the Ranked Spaces. II*

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In the preceding paper[1] we showed a method to construct outer measures in ranked spaces. But, in general, every open set is not always measurable. So in this note assuming some conditions we give a method to construct Borel measures in ranked spaces: By a Borel measure in an ω_0-ranked space R which satisfies F. Hausdorff's axiom (C) we mean a finite or infinite real valued, non-negative, and countably additive set function, defined on the countably additive class of sets, denoted by $\mathfrak{B}$, generated by the class of all open sets.

1. Definition 1. For two neighbourhoods $v(p)$ and $u(q)$ in an ω_0-ranked space we call that $v(p)$ is *strongly* contained in $u(q)$, denoted by $v(p)\Subset u(q)$, if there exists a neighbourhood $v'(p)$ of p such that $v(p)\subseteq v'(p)\subseteq u(q)$ and the rank of $v(p)>$the rank of $v'(p)>$the rank of $u(q)$. A disjoint finite family of neighbourhoods $\{v_n(p)\}$ is called a *packing* of a neighbourhood $u(q)$ if $v_n(p_n)\Subset u(q)$ for each n. And let $\{v_n(p_n)\}$ and $\{u_m(q_m)\}$ be two packings of $v(p)$ and $u(q)$ respectively. We call that the packings have the *same type* if, for each rank n, the number of neighbourhoods of rank n of $\{v_n(p_n)\}$ coincides with that of $\{u_m(q_m)\}$.

Let R be an ω_0-ranked space which satisfies the following conditions (1.1)-(1.4):[2]

(1.1) For every neighbourhood $v(p)$ of a point p there exists a rank n such that, for any rank m, $m\geq n$, there exists a neighbourhood $u(p)$ of rank m included in $v(p)$.[3]

(1.2) There is a rank n_0 such that, for any rank n, the upper limit of numbers of disjoint neighbourhoods of rank n contained strongly in a neighbourhood of rank n_0 is finite.[4]

(1.3) For two neighbourhoods $v(p)$ and $u(q)$ of the same rank and a packing of $v(p)$, $u(q)$ has a packing of the same type.[5]

(1.4) For any fundamental sequence $\{v_n(p_n)\}$ there exists a point p in $\bigcap_n v_n(p_n)$ such that, for any neighbourhood $v(p)$ of p, there exists

1) H. Okano: Measures in the ranked spaces, Proc. Japan Acad., **34**, 136-141 (1958), cited by [I] in this paper.

2) In the sequel we use the terminology *neighbourhood* only when it has a rank.

3) Cf. [I, (2.2)].

4) Cf. [I, (2.4)].

5) Cf. L. H. Loomis: Haar measure in uniform structure, Duke Math. J., **16**, 193-208 (1949).