58. Division Problem of Some Species of Distributions

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In this paper we shall show the following theorems. Theorem 1 was obtained by L. Ehrenpreis,*) but we give here a shorter proof.

Theorem 1. *Let Δ be any partial differential operator with constant coefficients. Then, for any distribution S there exists a distribution T such that $\Delta T=S$.*

Theorem 2. *Let Δ be as above. Then, for any distribution S of order k there exists a distribution T of order $k+2\left[\frac{n+2}{2}\right]+4$ such that $\Delta T=S$, where $[\quad]$ is the Gaussian symbol and n is the dimension of the underlying Euclidean space.*

Theorem 3. *Let Δ be as above. Then, for any locally square summable function f there exists a locally square summable function g such that $\Delta g=f$.*

1. **Preliminary notions.** By dx we denote the usual measure on R^n divided by $(2\pi)^{\frac{n}{2}}$. For any function $\varphi\in\mathcal{D}$, we define its Fourier transform $\Phi=\mathcal{F}(\varphi)$ by

$$\Phi(z)=\int\varphi(x)\,e^{-\sqrt{-1}x\cdot z}\,dx,$$

where $x\cdot z=x_1z_1+\cdots+x_nz_n$. We shall use lower case letters for functions of $\mathcal{D}$ and the corresponding upper case letters for their Fourier transforms.

Let us denote by D the set of all entire functions of exponential type which are rapidly decreasing on R^n. As is known, by the Paley-Wiener's theorem, D also can be characterized as the Fourier image of the set $\mathcal{D}$. We introduce a topology of D as follows. Let D_l be the set of all entire functions of exponential type $\leq l$ which are rapidly decreasing on R^n. Then $D=\bigcup_{l=1}^{\infty}D_l$. On D_l we give the topology defined by semi-norms

$$\nu_P(\Phi)=\sup_{z\in R^n}|P(z)\Phi(z)|,$$

where $P(z)$ denotes any polynomial on C^n. Then we can define the topology of inductive limit of the spaces D_l for $l=1,2,\cdots$. We give this topology on D. Then we can easily show that the Fourier transform is a topological isomorphism of $\mathcal{D}$ onto D.

*) L. Ehrenpreis: The division problem for distributions, Proc. Nat. Acad. Sci., **41**, 757-758 (1955); Solution of some problems of division, Amer. J. Math., **76**, 888-903 (1954).