83. *Representation of Some Topological Algebras. I*

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1. **Introduction.** In the study of the algebra of all continuous endomorphisms on a locally convex Hausdorff topological vector space, the abundance of endomorphisms of finite rank plays an important role. But, for an arbitrary algebra, there is of course no such a convenience in general, and it is interesting to determine an algebra which can be embedded into the algebra of all continuous endomorphisms on a locally convex vector space in such a manner that the embedded algebra contains every continuous endomorphism of finite rank. In this paper, we deal with this problem.

We shall be exclusively concerned with algebras over the complex or the real number field. A *topological algebra* is by definition an algebra and topological vector space such that the ring multiplication is separately continuous. Let E be an algebra; a topology with which E is a topological algebra is said to be *compatible* with the structure of E. As can readily be seen, we obtain the following proposition:

In order that a filter base $\mathfrak{B}$ on an algebra E is a fundamental system of neighbourhoods of 0 in E for a topology compatible with the structure of E, it is necessary and sufficient that $\mathfrak{B}$ possesses the following properties:[1]

1° For any number $\lambda \neq 0$, and for any $V \in \mathfrak{B}$, λV belongs to $\mathfrak{B}$.

2° Every member V of $\mathfrak{B}$ is absorbing, i.e. for any $x \in E$, there exists a number $\lambda \neq 0$ such that $\lambda x \in V$.

3° For any $U \in \mathfrak{B}$, there exists $V \in \mathfrak{B}$ such that $V+V \subseteq U$.

4° If $x \in E$, then for any $U \in \mathfrak{B}$, there exists $V \in \mathfrak{B}$ such that $xV \subseteq U$ and $Vx \subseteq U$.

Notice that we may assume $\mathfrak{B}$ to consist of circled sets.[2]

A topological algebra which is at the same time a locally convex topological vector space is called a *locally convex algebra*.

2. **Bounded sets.** Let E be a topological algebra. A subset A of E is called *left bounded* if for any neighbourhood V of 0 in E there exists a neighbourhood U of 0 in E such that $UA \subseteq V$. In an analogous way, we define right boundedness. A subset of E which

1) We employ the following notations: $\lambda A = \{\lambda x ;\ x \in A\}$, $aA = \{ax ;\ x \in A\}$, $AB = \{xy ;\ x \in A,\ y \in B\}$, $A+B = \{x+y ;\ x \in A,\ y \in B\}$.

2) A subset A of a vector space is said to be *circled* if $x \in A$ and $|\lambda| \leqq 1$ imply $\lambda x \in A$.