72. On Some Existence Theorems on Multiplicative Systems. I. Greatest Quotient[1)]

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§1. Introduction. In this paper we shall give the necessary and sufficient condition for a property P on multiplicative systems in order that for any system there exists its greatest P-quotient.

In the subsequent paper we shall study the similar problems for the existence of maximal P-subsystems.

We shall state here only definitions and main theorems, proofs of which will be omitted. We shall give the details in [1].

§2. Main theorems. A pair (S, M) is called a *multiplicative system* or simply *system*, if

(1) S is a set, and

(2) $M=\bigcup_{n=1}^{\infty}M_n$, M_n's are disjoint, and element of M_n is a mapping $S^n \to S$, and is called an *n-ary multiplication*.

In this paper we assume, in addition, S is not empty; even though later in the subsequent paper we will permit the null set as a system. Also M_n may be empty.

Let (S, M) and (S', M') be two systems and let $q: M \to M'$ be a one-to-one correspondence which sends M_n onto M'_n. A mapping $f: S \to S'$ is called a *homomorphism* if for any $m_n \in M$, and for any $a_1, a_2, \cdots, a_n \in S$, the equality $(a_1 f, a_2 f, \cdots, a_n f)\ (m_n q)=(a_1, a_2, \cdots, a_n) m_n f$ holds. A one-to-one onto homomorphism is an isomorphism.

Since q is a one-to-one correspondence, we can identify M' with M. In what follows we assume that all systems considered have the same set M of multiplications.

Now it is easy to define the notion of subsystems, quotients, natural homomorphisms, congruences, direct products, free products, free systems, etc.

Any one element system is isomorphic to any other, and it is called *trivial*.

Let $\{S_j : j \in J\}$ be a family of systems. Let p_j be the projection of the direct product $S=\Pi\{S_j : j \in J\}$ onto S_j. Then any subsystem S' of S such that $S'p_j=S_j$ for all $j \in J$ is called a *semi-direct product*[2)]

1) This work was supported in part by the National Science Foundation, U. S. A. This paper is an abstract of [1], which will be published elsewhere. Also see [2, pp. 1-52].

2) A semi-direct product defined here is a synonym for what is also called a subdirect product.