94. *Ideals in Non-commutative Lattices*

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(Comm. by K. KUNUGI, M.J.A., July 12, 1958)

§ 1. We published in 1953 a résumé of the theory of non-commutative lattices in C. R. Acad. Sci., Paris [11]. After this, we have received communications from Dr. F. Klein-Barmen and also from Prof. Dr. Pascual Jordan that Prof. P. Jordan with collaborators, Dr. E. Witt and Dr. W. Böge, had been constructing the theory of non-commutative lattices, independently of us, for the sake of applications in "theoretical physics" [3-6], and also independently, F. Klein published some excellent and interesting works on the similar articles [7-10].

Here, we shall make a survey of ideal theory in non-commutative lattices, from which the structure of some kinds of non-commutative lattices (normal and regular type) is decided (see § 3). This paper is also a résumé; and a full note, with complete proofs of [1], (titled "Theorie der nichtkommutativen Verbände I-II") will appear elsewhere.*)

§ 2. Let $\mathfrak{A}$ be an algebraic system with binary operators $*$ and $\circ$, both of which are associative and idempotent, but not necessarily commutative. If an order $<$ in $\mathfrak{A}$, i.e. i) $x<x$, ii) $x<y, y<x \rightarrow x=y$, iii) $x<y, y<z \rightarrow x<z$ for $x, y, z \in \mathfrak{A}$, satisfies a further condition: for any $a \in \mathfrak{A}$,

(2.1) $$x<y \rightarrow a*x<a*y \quad (\text{or } x*a<y*a),$$

then it is called left (resp. right) $*$-order of $\mathfrak{A}$. And if a left (or right) $*$-order $<$ of $\mathfrak{A}$ fulfils

(2.2) $$x<x*a \ (\text{resp. } x<a*x) \quad \text{for any } x, a \in \mathfrak{A},$$

then such $<$ is called a left (resp. right) *L-$*$-order* of $\mathfrak{A}$. Similarly, a left (or right) *L-$\circ$-order* of $\mathfrak{A}$ is defined.

Theorem 1. *In order that $\mathfrak{A}$ admit at least one left or right L-$*$-order, it is necessary and sufficient that the following equality be kept in $\mathfrak{A}$;*

(2.3) $$\alpha) \ x*a=x*a*x \quad \textit{resp.} \quad \beta) \ a*x=x*a*x.$$

An order $<$ (or $\prec$) is called *stronger* (resp. *weaker*) than $\prec$ (resp. $<$) if $a\prec b$ yields $a<b$: then

Theorem 2. *Suppose that $\mathfrak{A}$ satisfies the condition α) (or β)) in* (2.3) *above: Then*

I) *The order in $\mathfrak{A}$ defined by $a<b$ if and only if $b=a*x$ (resp.*

*) I express my hearty thanks to Dr. F. Klein-Barmen and also to Prof. Dr. Van der Waerden and his assistant Dr. R. Fischer for their precious advices and the precise examination of my theory.