# 116. *Finite-to-one Closed Mappings and Dimension. I*[1)]

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The fundamental theorem of this note is as follows.

**Theorem 1.** *Let $R$ and $S$ be metric spaces and $f$ a closed mapping (continuous transformation) of $R$ onto $S$. If $f^{-1}(y)$ consists of exactly $k(<\infty)$ points for every point $y \in S$ and* $\dim R \leq 0$, *then we have* $\dim S \leq 0$.[2)]

As direct consequences of this theorem we get a large number of theorems of dimension theory for non-separable metric spaces, among which there is Morita-Katětov's fundamental theorem of dimension theory. This fact indicates the possibility of the development of dimension theory, other than Morita and Katětov's, for non-separable metric spaces based on Theorem 1. An analogue to Theorem 1 for the case when $f$ is open will also be stated.

**Lemma 1.** *$R$ is a metric space with* $\dim R \leq 0$, *if and only if $R$ is a dense subset of an inverse limiting space of a sequence of discrete spaces.*

This is a trivial modification of Morita [2, Theorem 10.2] or of Katětov [1, Theorem 3.6]; its proof is included in that of Theorem 4 below.

*Proof of Theorem* 1. By Lemma 1 we can assume that $R$ is a dense subset of $\lim R_i$ obtained from $\{R_i, f_{jk}: R_j \to R_k\ (j>k)\}$ with discrete spaces $R_i = \{p_{i\alpha};\ \alpha \in A_i\}$. We can assume that points of $R_i$ are linearly-ordered such that for any $p_{i\alpha}, p_{i\beta}$ with $f_{ij}(p_{i\alpha}) \neq f_{ij}(p_{i\beta})$, $i>j$, it holds that $p_{i\alpha} > p_{i\beta}$ if and only if $f_{ij}(p_{i\alpha}) > f_{ij}(p_{i\beta})$. We introduce into points $(p_{1\alpha_1}, p_{2\alpha_2}, \cdots)$ of $\lim R_i$ the lexicographic order with respect to the one of $R_i$ just defined. Let $x_1(y), \cdots, x_k(y) \in R$ be the inverse image of $y \in S$ with $x_1(y) < \cdots < x_k(y)$ and then $R$ is decomposed into mutually disjoint subsets $T_i = \{x_i(y);\ y \in S\}$, $i=1, \cdots, k$.

We shall show that every $T_i$ is an $F_\sigma$. To do so it suffices to prove $T_1$ is an $F_\sigma$ since the rest case is proved similarly. Let $r(y)$, $y \in S$, be the smallest integer such that $\pi_r(x_1(y)), \cdots, \pi_r(x_k(y))$ are mutually different points of $R_r$, where $\pi_r: \lim R_i \to R_r$ is the natural projection. Let $S_t = \{y;\ y \in S,\ r(y) \leq t\}$, $t=1, 2, \cdots$, and $T_{jt} = T_j \cap f^{-1}(S_t)$ and then evidently i) $S = \bigcup_{t=1}^{\infty} S_t$, ii) $T_1 = \bigcup_{t=1}^{\infty} T_{1t}$, iii) $T_{1t} \subset T_{1,t+1}$. The

1) The detail of the content of the present note will be published in another place.
2) dim = covering dimension.