113. On Some Elementary Properties of the Crossed Products of von Neumann Algebras

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1. Many mathematical models would be employed by von Neumann in his investigation on the rings of operators. The theories of quantum mechanics, infinite groups, integration, measure preserving transformations and so on would be found among them. However, the classical theory of simple algebras would play an eminent rôle in his monumental "rings of operators" series, since the von Neumann algebras are recognized by himself as an infinite dimensional hypercomplex numbers in his earlier paper.

In this point of view, it is curious that the notion of crossed product is paid a little attension in the literatures, although it plays an essential rôle in the theory of simple algebras. As the authors concern, the first abstract definition of the crossed product of von Neumann algebras is introduced by Turumaru in 1955 by a seminar conversation, who also pointed out that the examples of factors due to Murray-von Neumann [2] is nothing but the crossed product of an abelian algebra by an ergodic automorphism group. However, the further development delayed, since no security existed that the crossed product of a factor produces an another new one.

In the succeeding paper [3], it will be proved that the crossed product of the hyperfinite continuous factor is not isomorphic to the original when the group of automorphisms is suitably restricted. Therefore, the purpose of the present note is to develope some elementary properties of the crossed product of a finite factor along the line that some well-known theorems of simple algebras are still valid for finite factors. Incidentally, in the end of the note, a theorem of Murray-von Neumann concerning the example construction will be given an alternative simpler proof basing on the same idea.

2. At the beginning, we may make a few remarks. The terminology of J. Dixmier [1] will be used without any explanation unless the contrary is stated (for example, von Neumann algebra and the hyperfiniteness will be used instead of W^*-algebras and the approximate finiteness respectively). Moreover, each von Neumann algebra of the present note will be assumed to act on a separable Hilbert space.