108. *Hukuhara's Problem for Hyperbolic Equations with Two Independent Variables. II. Quasi-linear Case*

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1. **Introduction.** In Part I of this report, we have explained the concept of Hukuhara's problem (we shall use the abbreviation "Problem H" hereafter) for partial differential equations, and proved its correct posedness for the semi-linear hyperbolic systems with two independent variables. In this part, we show the same results for the quasi-linear system.

Consider the real quasi-linear system of the form

(1) $\partial u_i/\partial t-\lambda_i\ (t, x, u)\cdot\partial u_i/\partial x=f_i\ (t, x, u),\quad i=1, 2, \cdots, N,$

where u stands for the N-dimensional vector $(u_1, \cdots, u_N)$. We adopt the notations λ, f, etc. to represent the vectors $(\lambda_1, \cdots, \lambda_N)$, $(f_1, \cdots, f_N)$, etc. We define the norm $\|g\|_D$ of a vector $g=(g_1, \cdots, g_N)$ whose components g_i are functions defined on some domain D, such as $\|g\|_D=\sup_i[\sup_D|g_i|]$.

Our Problem H for the quasi-linear system (1) is defined quite similarly to the semi-linear case. Namely, let N curves C_i, $i=1, 2, \cdots, N$ be given on a strip $G_0=\{0\leq t\leq B_0, B_0>0\}$ of (t, x)-space and let the value of the i-th unknown u_i of (1) be prescribed on the i-th curve C_i for $i=1, 2, \cdots, N$. Under these conditions we shall study the equations (1).

If we impose certain restrictions on the magnitude of the constant B_0 and on the situation of the curves C_i, then the Problem H has a solution which is unique and stable under a certain class of smooth functions.

1° We assume that the components of λ and f are defined and continuous on a strip $\overline{G}=\{0\leq t\leq B, \|u\|\leq\rho;\ B, \rho>0\}$ of (t, x, u)-space, where $\|u\|$ is defined such as $\|u\|=\mathrm{Sup}_i|u_i|$. We assume further that they have continuous derivatives up to second order with respect to u, x and their mixed differentiation, and that the norms $\|\cdot\|_{\overline{G}}$ of all those derived vectors including λ and f themselves are finite.

2° Conditions for the curves C_i

We require that for every i, the i-th curve C_i should be twice continuously differentiable, the absolute value of its curvature should be less than a constant Γ, and C_i should be *uniformly transversal* to the whole family of the i-th characteristics of (1) for every u such as $\|u\|\leq\rho$. The explanation of the last statement is as follows. Let