107. A Remark on My Paper "A Boundary Value Problem of Partial Differential Equations of Parabolic Type" in Duke Mathematical Journal

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§1. **Introduction.** Recently Dr. T. Shirota kindly called the author's attention to the following fact;—in the author's paper [1] published in Duke Mathematical Journal, 24, the continuity of $p_z(t, x; s, y)$—and accordingly that of the fundamental solution $u(t, x; s, y)$—in $y \in \boldsymbol{B}$ is not obvious in the case where $\alpha(t, \xi)$ takes the value zero for some $\langle t, \xi \rangle$ and is not identically zero. The same situation occurs in the author's another paper [2]. In the present note, instead of completing the proof of the continuity of the fundamental solution, we shall slightly modify the argument in [1].

The argument in the present note may be adapted to [2]. By the way, we state the following correction to the paper [2];—$[1-p_z(s, y; t, x)]$ *in the numerator of the right-hand side of* (3.24) *in* [2, p. 63] *should be replaced by* $p_z(s, y; t, x)$.

§2. **Construction of the fundamental solution.** We shall use notations stated in [1] without repeating definitions of them. We first notice that, if $\alpha(t, \xi)$ identically equals zero or is bounded away from zero, $p_z(t, x; s, y)$ has desired regularity and accordingly $u(t, x; s, y)$ does.

For each $n \geq 1$, let $\chi_n(\lambda)$ be a monotone increasing function of class C^3 in $\lambda \in [0, 1]$ such that

$$(1) \qquad \begin{aligned} \chi_n(\lambda) &= 1/(n+1) \qquad \text{for } \lambda \leq 1/(n+2) \\ &= \lambda \qquad\qquad\quad \text{for } \lambda \geq 1/n. \end{aligned}$$

We define $\alpha_n(t, \xi)$ and $\beta_n(t, \xi)$ on $[s_0, t_0] \times \boldsymbol{B}$ for $n=0, 1, 2, \cdots$ as follows:

$$(2) \qquad \begin{cases} \alpha_0(t, \xi)=0, \quad \alpha_n(t, \xi)=\chi_n(\alpha(t, \xi)) \quad (n \geq 1) \\ \text{and} \quad \beta_n(t, \xi)=1-\alpha_n(t, \xi) \quad (n=0, 1, 2, \cdots) \end{cases}$$

where $\alpha(t, \xi)$ is the function stated in the given boundary condition (B_φ^t) in [1]. Then, for each $n \geq 0$, we may apply the argument in [1] to the parabolic equation $Lf+h=0$ associated with boundary condition:

$$(B_{n,\varphi}^t) \qquad \alpha_n(t, \xi)f(t, \xi)+\beta_n(t, \xi)[\partial f(t, \xi)/\partial \boldsymbol{n}_{t,\xi}]=\varphi(t, \xi),$$

and obtain the fundamental solution $u_n(t, x; s, y)$ with all properties stated in [1] where (B_φ^t) is replaced by $(B_{n,\varphi}^t)$.

Let $f(x)$ be an arbitrary continuous and non-negative function on $\overline{\boldsymbol{D}}$ and put