139. *On the Cohomology Groups of $\mathfrak{p}$-adic Number Fields*

By Hideo KUNIYOSHI

Mathematical Institute, Tôhoku University

(Comm. by K. SHODA, M.J.A., Nov. 12, 1958)

In the present note we shall study the cohomology groups of the ring of all $\mathfrak{p}$-integers of a $\mathfrak{p}$-adic field.

Let K be a $\mathfrak{p}$-adic number field and let L be a finite separable extension field over K. More generally, let K be a complete field by a discrete valuation and let L be a finite separable extension field over K with separable residue class field. Let R and Λ be the rings of all $\mathfrak{p}$-integers of K and L, respectively. Then Λ has a minimal basis over R, i.e.

$$\Lambda=R+R\theta+\cdots+R\theta^{n-1},$$

where $1, \theta, \cdots, \theta^{n-1}$ are linearly independent over R [1]. Let $f(x)=0$ be the equation of θ in R.

We shall consider Λ as an algebra over R and construct a Λ^e-projective resolution over Λ which is suitable for our purpose.

Let

$$f(x)=(x-\theta)g(x), \quad g(x)=x^{n-1}+(\sum_j b_{n-2,\,j}\,\theta^j)x^{n-2}+\cdots$$

be the decomposition of $f(x)$ in Λ. We put

$$g_e(\theta)=\sum_{i,j} b_{ij}\,\theta^i\otimes\theta^j \quad {}^{1)}$$

$$\Delta\theta=\theta\otimes 1-1\otimes\theta$$

in $\Lambda^e=\Lambda\otimes_R\Lambda$.

Lemma

Let $\sum\lambda\otimes\mu$ *be any element in* Λ^e. *Then*

$(\sum\lambda\otimes\mu)(\theta\otimes 1-1\otimes\theta)=0$ *if and only if* $\sum\lambda\otimes\mu\in\Lambda^e\cdot g_e(\theta)$;

$(\sum\lambda\otimes\mu)\cdot g_e(\theta)=0$ *if and only if* $\sum\lambda\otimes\mu\in\Lambda^e(\theta\otimes 1-1\otimes\theta)$.

Proof. Since we have a ring isomorphism

$$\Lambda\otimes_R\Lambda\cong\Lambda[x]/(f(x)),$$

$$\theta\otimes 1-1\otimes\theta \longleftrightarrow x-\theta \mod (f(x)),$$

$$g_e(\theta) \longleftrightarrow g(x) \mod (f(x)),$$

we shall calculate in the right hand side. We take polynomials of degree less than n as the uniquely determined representatives of the classes mod $f(x)$. If $(x-\theta)h(x)\equiv 0 \mod f(x)$, deg $h(x)\leqq n-1$, then dividing $h(x)$ by $g(x)$ we have $h(x)=\alpha g(x)+s(x)$, deg $s(x)\leqq n-2$; so $s(x)(x-\theta)\equiv 0 \mod f(x)$. Therefore $s(x)=0$, $h(x)=\alpha g(x)$. Similarly, if $g(x)h(x)\equiv 0 \mod f(x)$, then $h(x)=(x-\theta)h_0(x)$.

Lemma

1) Since Λ is commutative, $\Lambda^*\cong\Lambda$ and we shall drop the sign $*$.