156. A Generalization of Vainberg's Theorem. II

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3. Semi-ordered linear spaces R and R' are said to be *similar to each other* if there exists a one-to-one correspondence $\varphi: R \ni a \rightarrow \varphi(a) \in R'$ between R and R' such that

(3.1) $\varphi(-a) = -\varphi(a)$ for all $a \in R$;

(3.2) $\varphi(a) \geq \varphi(b)$ if and only if $a \geq b$.

The correspondence φ fulfilling the conditions (3.1), (3.2) is called a similar correspondence.

A convex set C in R is said to be an *l-vicinity* if

(3.3) *for any* $a \in R$, *there exists a positive number* α *such that* $\alpha a \in C$;

(3.4) $a \in C$, $|b| \leq |a|$ *implies* $b \in C$;

(3.5) $a, b \in C$, $|a| \frown |b| = 0$ *implies* $a + b \in C$.

If C is a convex l-vicinity then we have $0 \in C$ and $a \asymp b \in C$ for any $a, b \in C$.

Now we say that semi-ordered linear spaces R and R' are *almost similar to each other*, if there exist convex l-vicinities $C \subset R$, $C' \subset R'$ and a similar correspondence ψ from C onto C'. For such ψ we have obviously for $a, b \in C$

$$\psi(a \asymp b) = \psi(a) \asymp \psi(b), \quad \psi(|a|) = |\psi(a)|.$$

When R and R' are almost similar to each other, then for any normal manifold N in R (projection operator $[N]$ on R) there exists a normal manifold in R' (resp. a projection operator $[N]'$ on R') such that

$$x \in [N]C \quad \text{if and only if} \quad \psi(x) \in [N]'C'.$$

Therefore we can conclude that *the proper space* $\mathcal{E}$[1] *of* R *is homeomorphic to that* $\mathcal{E}'$ *of* R', *if* R *and* R' *are almost similar to each other.* The converse of this fact, however, is not true in general. But as for modulared semi-ordered linear spaces we can show *the converse of the above holds valid in sufficiently general cases.* This gives appropriateness for our standpoint of discussing the theme of this paper in modulared semi-ordered linear spaces. The proof of the following theorem owes essentially its idea to that of Theorem 62.1 in [1].

1) In fact, $[N]'$ is a projection operator on R' defined by the least normal manifold including all $\psi(x)$ $(x \in [N]C)$.