155. *Remarks on Some Riemann Surfaces*

By Kikuji MATSUMOTO
Mathematical Institute, Nagoya University
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1. In the theory of meromorphic functions, it is important to investigate the properties of covering surfaces generated by their inverse functions. For this purpose, the study of properties of a non-compact region of a Riemann surface is useful.

Recently Kuramochi [4] gave interesting results for non-compact regions of some Riemann surfaces and these results were extended by Constantinescu and Cornea [1] and himself [5]. On the other hand, the method given by Heins [2] is also expected to contribute for the same purpose. So, in this note, we shall investigate some properties of covering surfaces using Kuramochi's results and Heins' method. Here we shall omit the details which will appear elsewhere.

2. Let R_1 and R_2 be two Riemann surfaces which do not belong to O_G, and let f be a conformal mapping of R_1 into R_2. We denote by $\mathfrak{G}_{R_1}$ and $\mathfrak{G}_{R_2}$ Green functions of R_1 and R_2 respectively. Then, holds the equality

$$\mathfrak{G}_{R_2}(f(p); q) = \sum_{f(r)=q} n(r)\mathfrak{G}_{R_1}(p; r) + u_q(p),$$

where $n(r)$ is the multiplicity of f at $r \in R_1$, and $u_q(p)$ is the greatest harmonic minorant of $\mathfrak{G}_{R_2}(f(p); q)$ on R_1.

Generally, a positive harmonic function is representable uniquely by the sum of a positive quasi-bounded harmonic function and a positive singular harmonic function (Parreau [7]). Heins [2] proved that $u_q(p)$ is quasi-bounded except for a set of q of capacity zero and that the quasi-bounded component of $u_q(p)$ is either positive on $R_1 \times R_2$ or constantly zero.

According to Heins [2], we say that f is of type-Bl if the second alternative occurs for f.

Now, let R_1 and R_2 be arbitrary Riemann surfaces, and let f be a conformal mapping of R_1 into R_2. We shall say that f is of type-Bl at $q \in R_2$ provided that there exists a simply-connected Jordan region Ω satisfying: (1) $q \in \Omega \subset R_2$, (2) $f^{-1}(\Omega) \neq \phi$ and (3) for each component Δ of $f^{-1}(\Omega)$, the restriction f_Δ of f to Δ is of type-Bl considering f_Δ as to be a conformal mapping of Δ into Ω. We shall say that f is locally of type-Bl if f is of type-Bl at each point of R_2.

For simplicity, we shall call a non-compact or compact domain on a Riemann surface R a subregion on R when its relative boundary with respect to R consists of at most an enumerable number of