153. *Homomorphisms of a Left Simple Semigroup onto a Group*

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Cohn, in his paper [1], defined d-semigroups as semigroups S satisfying the following conditions:

(1) if $a, b \in S$, then $a = xb$ for some $x \in S$,

(2) if $a, b \in S$, then either $a = b$ or $a = by$ or $b = ay$ for some $y \in S$,

(3) S contains no idempotent,

and then he characterized the kernels of homomorphisms of a d-semigroup onto a group.

In this note, we show that a similar result holds for left simple semigroups, that is, semigroups satisfying the condition (1) only.

In this note, S *denotes always a left simple semigroup.*

A subsemigroup T of S is said to be *left unitary* in S, if T contains, with any a, b, all solutions x in S of the equation $ax = b$. (This definition is due to Dubreil [2]. Cohn uses the word 'closed' in the sense of 'right and left unitary'.) Also, a subsemigroup T of S is said to be *normal* in S, if $xT \subseteq Tx$ for any $x \in S$.

In S, we define a set U by

$$U = \{x \in S;\ xa = a \text{ for some } a \in S\}.$$

U is non-empty, since S satisfies the condition (1). Also, we define a set V by

$$V = \{x \in S;\ ux = u' \text{ for some } u, u' \in U\}.$$

Lemma 1. $U \subseteq V$.

Proof. If $u \in U$, there exists an element $a \in S$ such that $ua = a$. Then we have also $u^2 a = ua = a$, and so $u^2 \in U$. But u is a solution of the equation $ux = u^2$ and so we have $u \in V$.

By Lemma 1, V is also non-empty.

Now we consider the subsemigroup I generated by the set V, and call it the *core* of S. Thus every element of the core I can be represented by a finite product of elements in V.

Lemma 2. *Given $x \in S$ and $v \in V$, there exists an element $v' \in V$ such that $xv = v'x$.*

Proof. By the condition (1), there exists an element $v' \in S$ such that $xv = v'x$. Since $v \in V$, there exist two elements $u_1, u_2 \in U$ such that $u_1 v = u_2$. Then, since $u_1, u_2 \in U$, there exist elements $a, b \in S$ such that $u_1 a = a$ and $u_2 b = b$. Using the condition (1) again, we can consider an element $s \in S$ such that $x = su_1$, and then an element $p \in S$ such that