152. *Note on Fundamental Exact Sequences in Homology and Cohomology for Non-normal Subgroups*

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The purpose of the present note is to observe that the fundamental exact sequences, or the exact sequences of Hochschild-Serre [4], in homology and cohomology of groups, which describe a certain relationship between homology or cohomology groups of a group, its normal subgroup, and the factor group, may be extended to the case of non-normal subgroups.

Thus, let G be a group and H a subgroup of G. With a (left) G-module M, Adamson [1] defines relative cohomology groups $H^n([G, H], M)$ on M, which in case H is normal in G turn out to coincide with the ordinary cohomology groups $H^n(G/H, M^H)$ of the factor group $G/H, M^H$ being the submodule of M consisting of all elements of M left invariant by H. The relative cohomology groups $H^n([G, H], M)$ may be defined either in terms of the standard complex for $[G, H]$, as in [1], or more generally in terms of any $[G, H]$-projective resolution of the module Z of rational integers (i.e. a $(Z[G], Z[S])$-exact sequence $0 \leftarrow Z \leftarrow X_0 \leftarrow X_1 \leftarrow \cdots$ of $Z[G]$-modules in which each X_i is $(Z[G], Z[S])$-projective), and may be expressed as $\mathrm{Ext}^n_{[G, H]}(Z, M)$ $(=\mathrm{Ext}^n_{(Z[G], Z[H])}(Z, M))$, in the terminology and notation of Hochschild [3]. Now, Adamson [1] proves that if here $H^m(U, M)=0$ for $m=1, \cdots, n-1$ $(n>0)$ and for every subgroup U of G which is an intersection of conjugates of H then the sequence

$$0 \rightarrow H^n([G, H], M) \xrightarrow{\lambda} H^n(G, M) \xrightarrow{\rho} H^n(H, M)$$

is exact, where ρ is the ordinary restriction map and λ is the lifting (or inflation) map defined for instance by the natural map of the standard complex of G onto that of $[G, H]$. We contend that this exact sequence can be enlarged to a larger exact sequence which specializes to the exact sequence of Hochschild-Serre [4] in case H is normal in G. Thus, under the same assumption as above, $H^m(U, M)=0$ for $m=1, \cdots, n-1$ $(n>0)$ and for every subgroup U of G which is an intersection of conjugates of H, we have an exact sequence

$$(1) \qquad \begin{aligned} 0 \rightarrow H^n([G, H], M) &\xrightarrow{\lambda} H^n(G, M) \xrightarrow{\rho} H^n(H, M)^I \\ &\xrightarrow{\tau} H^{n+1}([G, H], M) \xrightarrow{\lambda} H^{n+1}(G, M), \end{aligned}$$

where the maps λ, ρ are as before, $H^n(H, M)^I$ is a certain subgroup of $H^n(H, M)$, and the map τ, transgression, is defined, similarly as in