150. *On the Singular Integrals. IV* [*)]

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1. This is a continuation of the previous paper [4, III]. The purpose of this paper is to show the reciprocal formula of the Hilbert operator. The method of proof is a so-called complex variable method which is different from that of the previous one quitely. As an application, we can establish some results for analytic functions in a half-plane.

Let $g(x)$ be a real valued measurable function over $(-\infty, \infty)$ we put

$$C(z, g)=\frac{1}{2\pi i}\int_{-\infty}^{\infty} g(t)\frac{dt}{t-z}, \tag{1.1}$$

$$P(z, g)=\frac{1}{\pi}\int_{-\infty}^{\infty} g(t)\frac{y\,dt}{(t-x)^2+y^2} \tag{1.2}$$

$$\widetilde{P}(z, g)=-\frac{1}{\pi}\int_{-\infty}^{\infty} g(t)\frac{(t-x)dt}{(t-x)+y^2}. \tag{1.3}$$

We shall call $C(z, g)$ and $P(z, g)$ integrals of Cauchy type and Poisson type respectively, associated with the function $g(x)$. We observe also that

$$2C(z, g)=P(z, g)+i\widetilde{P}(z, g). \tag{1.4}$$

We have then

Theorem 1. *Let $g(x)$ belong to L_μ^p $(p\geqq 1, 0\leqq\alpha<1)$; then we have*

$$\text{(S)-}\lim_{y\to 0} P(z, g)=g(x), \quad a.e. \tag{1.5}$$

$$\lim_{y\to 0}\int_{-\infty}^{\infty}\frac{|P(z, g)-g(x)|^p}{1+|x|^\alpha}dx=0, \tag{1.6}$$

where the sign (S) *means that the limit exists along a Stoltz' path—as an angular limit.*

Theorem 2. *Let $g(x)$ belong to L_μ^p $(p>1, 0\leqq\alpha<1)$ or $g(x)$ and $\widetilde{g}(x)$ both belong to L_μ $(0\leqq\alpha<1)$. Then we have also*

$$\text{(S)-}\lim_{y\to 0}\widetilde{P}(z, g)=\widetilde{g}(x), \quad a.e. \tag{1.7}$$

$$\lim_{y\to 0}\int_{-\infty}^{\infty}\frac{|\widetilde{P}(z, g)-\widetilde{g}(x)|^p}{1+|x|^\alpha}dx=0. \tag{1.8}$$

For this purpose it is enough to prove

Theorem 3. *Under the assumption of Theorem* 2 *we have*

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*) Here we state the result without proof. The detailed argument will appear in Jour. Fac. Sci. Hokkaidô University.