149. On Convolution of Laurent Series

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1. Related to a conjecture proposed by Pólya and Schoenberg [4], we have observed in a previous paper [1] a class $\mathfrak{R}_0$ of regular analytic functions defined in the unit circle $|z|<1$ which are of positive real part there and equal to unity at the origin. It has been shown that if both functions

$$f(z)=1+2\sum_{n=1}^{\infty}a_nz^n \quad \text{and} \quad g(z)=1+2\sum_{n=1}^{\infty}b_nz^n$$

belong to $\mathfrak{R}_0$ then the function defined by

$$h(z)=1+2\sum_{n=1}^{\infty}a_nb_nz^n$$

also belongs to $\mathfrak{R}_0$.

In the same paper [1], we have also observed, as a straightforward generalization of the class $\mathfrak{R}_0$, a class $\mathfrak{R}_q$ of single-valued regular analytic functions defined in an annulus $(0<)q<|z|<1$ which are of positive real part and normalized by the conditions that their values on $|z|=q$ have the constant real part and that their Laurent expansions have the constant term equal to unity. For this class, it has been shown that if both functions

$$f(z)=1+2\sum_{n=-\infty}^{\infty}{}' \frac{a_n}{1-q^{2n}}z^n \quad \text{and} \quad g(z)=1+2\sum_{n=-\infty}^{\infty}{}' \frac{b_n}{1-q^{2n}}z^n$$

belong to $\mathfrak{R}_q$ then the function defined by

$$h(z)=1+2\sum_{n=-\infty}^{\infty}{}' \frac{a_nb_n}{1-q^{2n}}z^n$$

also belongs to $\mathfrak{R}_q$; here the prime means that the summand with the suffix $n=0$ is to be omitted.

On the other hand, in a previous paper [2], we have considered, together with the classes mentioned above, a wider class $\hat{\mathfrak{R}}_q$ which is obtained by rejecting the restricting condition for $\mathfrak{R}_q$ imposed on image of $|z|=q$. Namely, the class consists of single-valued regular analytic functions defined in an annulus $(0<)q<|z|<1$ which are of positive real part and normalized by the condition that their Laurent expansions have the constant term equal to unity.

The result on $\mathfrak{R}_q$ referred to above does not admit a formally direct generalization for the class $\hat{\mathfrak{R}}_q$ as it stands. In fact, for functions