6. *Convergence Concepts in Semi-ordered Linear Spaces. I*

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Concerning semi-ordered linear spaces, L. Kantorovitch [1] gave originally two different concepts of convergence, that is, order convergence and star convergence. One of the authors introduced two other concepts, that is, dilatator convergence in [2] and individual convergence in [3], which are essentially equivalent to each other. Combining these concepts, we also obtain star-individual convergence in [4]. In this paper we want to discuss these concepts of convergence and their combinations more systematically. In the sequel we will use the terminologies and notations in the book [4].

Let R be a continuous semi-ordered linear space. We consider the order convergence basic, that is, for a sequence $a_\nu \in R$ ($\nu=0,1,2,\cdots$), $a_0=\lim_{\nu\to\infty} a_\nu$ means

$$a_0=\bigcap_{\nu=1}^{\infty}\bigcup_{\mu\geqq\nu} a_\mu=\bigcup_{\nu=1}^{\infty}\bigcap_{\mu\geqq\nu} a_\mu.$$

In the sequel we denote by $\{a_\nu\}_\nu$ an arbitrary sequence $a_\nu\in R$ ($\nu=0,1,2,\cdots$) and $\{a_\nu\}_{\nu\geqq 1}$ means a_ν ($\nu=1,2,\cdots$). A mapping $\mathfrak{a}$ of all sequences $\{a_\nu\}_\nu$ to sequences $\{a_\nu^{\mathfrak{a}}\}_\nu$ is called an *operator*, if

1) $\quad a_0=\lim_{\nu\to\infty} a_\nu$ implies $a_0^{\mathfrak{a}}=\lim_{\nu\to\infty} a_\nu^{\mathfrak{a}}$,

2) $\quad \{a_\nu^{\mathfrak{a}}\}_{\nu\geqq 1}$ depends only upon $\{a_\nu\}_{\nu\geqq 1}$

that is, $a_\nu=b_\nu$ ($\nu=1,2,\cdots$) implies $a_\nu^{\mathfrak{a}}=b_\nu^{\mathfrak{a}}$ ($\nu=1,2,\cdots$). An operator $\mathfrak{a}$ is said to be *linear* if

$$(\alpha a_\nu+\beta b_\nu)^{\mathfrak{a}}=\alpha a_\nu^{\mathfrak{a}}+\beta b_\nu^{\mathfrak{a}} \qquad (\nu=0,1,2,\cdots).$$

For two operators $\mathfrak{a},\mathfrak{b}$, putting

$$a_\nu^{\mathfrak{a}\mathfrak{b}}=(a_\nu^{\mathfrak{a}})^{\mathfrak{b}} \qquad (\nu=0,1,2,\cdots),$$

we also obtain an operator $\mathfrak{a}\mathfrak{b}$, which will be called the *product* of $\mathfrak{a}$ and $\mathfrak{b}$. With this definition, we have obviously

$$(\mathfrak{a}\mathfrak{b})\mathfrak{c}=\mathfrak{a}(\mathfrak{b}\mathfrak{c}).$$

$\mathfrak{a}$ is said to *commute* $\mathfrak{b}$, if $\mathfrak{a}\mathfrak{b}=\mathfrak{b}\mathfrak{a}$.

A set $\mathfrak{A}$ of operators is called a *process*, if for any two sequences $\{a_\nu\}_\nu$, $\{b_\nu\}_\nu$ with $a_0\neq b_0$ we can find $\mathfrak{a}\in\mathfrak{A}$ for which $a_0^{\mathfrak{a}}\neq b_0^{\mathfrak{a}}$. A set A of processes is called a *modificator*, if for any $\mathfrak{A}_1,\mathfrak{A}_2\in A$ we can find $\mathfrak{A}\in A$ for which $\mathfrak{A}\subset\mathfrak{A}_1,\mathfrak{A}_2$. For two modificators A,B we write $A\geqq B$, if for any $\mathfrak{A}\in A$ we can find $\mathfrak{B}\in B$ for which $\mathfrak{A}\supset\mathfrak{B}$. If $A\geqq B$ and $B\geqq A$ at the same time, we write $A=B$.

Let A and B be modificators. For a process $\mathfrak{A}\in A$ and a system of processes $\mathfrak{B}_{\mathfrak{a}}\in B$ ($\mathfrak{a}\in\mathfrak{A}$) we see easily that the set

$$\{\mathfrak{a}\mathfrak{b}:\ \mathfrak{a}\in\mathfrak{A},\ \mathfrak{b}\in\mathfrak{B}_{\mathfrak{a}}\}$$