2. *Notes on Tauberian Theorems for Riemann Summability. II*

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In this note we shall deal with the problem proposed in §12 of Yano [6]. We prove a theorem (Theorem 1) concerning Riemann summability by using Lemma 3. Riemann summability of $\sum a_n$ is closely connected with Cesàro summability of an even function $\varphi(t)\in L$ with Fourier coefficients a_n. Here we notice that in Riemann summability a_n are independent of Fourier coefficients. Lemma 1 will interpret the relation between these two summabilities by the help of Lemmas 2 and 4; — this is a chief object of this paper. In §3 we shall give "Riemann-Cesàro summability"—analogue.

1. Riemann summability. A series

$$\sum a_\nu=\sum_{\nu=1}^{\infty} a_\nu \quad (a_0=0)$$

is said to be summable to sum s by Riemann method of order p, or briefly summable (R, p) to s, if the series in

$$F(t)=\sum_{\nu=1}^{\infty} a_\nu\left(\frac{\sin \nu t}{\nu t}\right)^p$$

converges in some interval $0<t<t_0$, and $F(t)\to s$ as $t\to 0$ (cf. Verblunsky [1]). Here we suppose that p is a positive integer, and a_n are real throughout this paper.

The n-th Cesàro sum of order r of $\sum a_\nu$ is

$$s_n^r=\sum_{\nu=0}^{n} A_{n-\nu}^r\, a_\nu \qquad (-\infty<r<\infty),$$

where A_n^r is defined by the identity

$$(1-x)^{-r-1}=\sum_{n=0}^{\infty} A_n^r x^n \qquad (|x|<1),$$

and in particular $a_n=s_n^{-1}$.

THEOREM 1. Let $-1\leqq b$,*) $b<p-1<\gamma<\beta$, and $\delta=\dfrac{p-1-b}{\beta-p+1}(\beta-\gamma)$. If

$$\sum_{\nu=1}^{n} |s_\nu^\beta|=o(n^{\gamma+1}) \tag{1.1}$$

$$\sum_{\nu=n}^{2n} (|s_\nu^b|-s_\nu^b)=O(n^{b+\delta+1}) \tag{1.2}$$

as $n\to\infty$, then $\sum a_\nu$ is summable (R, p) to zero.

In the case $b=-1$ we have the following corollary.

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*) We could remove the restriction $b\geqq -1$ in this theorem by the argument used in Yano [5].