29. On Schlicht Functions. I

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It is known that $w=\frac{1+z}{1-z}$ is a schlicht (convex) function with a positive real part for $|z|<1$, and, when $|z|=1$, w corresponds to the imaginary axis. Hence, for $|z|<1$, $\left(\frac{1+z}{1-z}\right)^2$ is a schlicht function with the cut on the negative real axis. For any real number λ, a function

$$\left[\frac{1+z}{1-z}+i\lambda\right]^2; \quad |z|<1$$

is univalent, and for any positive number $\mu\geqq 0$,

$$\left\{\left[\frac{1+z}{1-z}+i\lambda\right]^2+\mu\right\}^{\frac{1}{2}}; \quad |z|<1$$

is a schlicht function with positive real part. For any two real numbers λ_1 and λ_2,

$$\left[\left\{\left[\frac{1+z}{1-z}+i\lambda_1\right]^2+\mu\right\}^{\frac{1}{2}}+i\lambda_2\right]^2$$

is univalent for $|z|<1$.

In such a way, we can form a class of schlicht functions which have a certain type of slits. That is, for any set of real numbers $\lambda_1, \lambda_2,\cdots, \lambda_k$, and for any positive numbers $\mu_1, \mu_2,\cdots, \mu_{k-1}$, a function defined by

$$(1) \qquad \left[\cdots\left\{\left[\left\{\left(\frac{1+z}{1-z}+i\lambda_1\right)^2+\mu_1\right\}^{\frac{1}{2}}+i\lambda_2\right]^2+\cdots+\mu_{k-1}\right\}^{\frac{1}{2}}+i\lambda_k\right]^2$$

is analytic and univalent for $|z|<1$, and the values form a region with a tree-shaped slit.

The chief object of this paper is to give properties of coefficients obtained by Taylor expansion of such a function.

Let $F_k(z)$ be denoted by the function (1), we have

$$(2) \qquad \begin{cases} F_0(z)=\dfrac{1}{(1-z)^2}(1+z)^2\equiv\dfrac{1}{(1-z)^2}[\varphi_0(z)]^2 \\ F_1(z)=\dfrac{1}{(1-z)^2}[1+z+i\lambda_1(1-z)]^2\equiv\dfrac{1}{(1-z)^2}[\varphi_1(z)]^2 \\ \cdots\cdots\cdots\cdots\cdots \\ F_k(z)=\dfrac{1}{(1-z)^2}[\cdots\{[1+z+i\lambda_1(1-z)]^2+\mu_1(1-z)^2\}^{\frac{1}{2}}+\cdots \\ \qquad +i\lambda_k(1-z)]^2\equiv\dfrac{1}{(1-z)^2}[\varphi_k(z)]^2 \\ \cdots\cdots\cdots\cdots\cdots . \end{cases}$$