26. On Semi-continuity of Functionals. II

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1. Introduction. In earlier paper [2], we have proved Theorem 1 [2] which is concerned with the semi-continuity of additive functionals on semi-ordered linear spaces. By the same notion, we shall obtain some results concerning additive functionals on Boolean algebras.[1] Let B be a σ-complete[2] Boolean algebra. A positive functional m on B is called a *finitely additive measure* if the following condition is satisfied.

(1.1) $$m(x+y)=m(x)+m(y) \quad \text{for } x, y \in B \quad \text{with } x \frown y=0.$$

Furthermore if the functional m satisfies the following condition (1.2), m is called a *totally additive measure.*

(1.2) *For a system of mutually orthogonal elements* x_i $(i=1, 2,\cdots)$ *we have*

$$m(\bigcup_{i=1}^{\infty} x_i) = \sum_{i=1}^{\infty} m(x_i)$$

(1.2) implies (1.1), but the converse does not follow. However, sometimes a finitely additive measure is totally additive on some ideal[3] of B.

If B is a Boolean algebra, then we can consider the representation space. (This space consists of all dual maximal ideals $\mathfrak{p}$ of B.) We denote this space by $\mathfrak{E}$. $\mathfrak{E}$ constitutes a compact Hausdorff space with open basis: $U_x=\{\mathfrak{p}:\mathfrak{p}\ni x\}$, $x\in B$.

If B is σ-complete, then the closure of a σ-open set (countable union of closed sets) of $\mathfrak{E}$ is open in $\mathfrak{E}$. An ideal I of B is said to be *dense* in B if for any $x(\neq 0)\in B$ there exists an element $y\in I$ with $0\neq y\leqq x$.

We shall consider the following property of σ-complete Boolean algebra.

(A) *Let* A_n $(n=1, 2,\cdots)\subset\mathfrak{E}$ *be* σ*-open and dense. Then we can find an open dense set* $U\subset\mathfrak{E}$ *with* $U\subset\bigcap_{n=1}^{\infty} A_n$.

We have also the following property equivalent to (A).

(A′) *Let* B_n $(n=1, 2,\cdots)\subset\mathfrak{E}$ *be* δ*-closed*[4] *and no-where dense*

1) For the definition of Boolean algebra, see [1, Chapter 10].
2) B is σ-complete if for x_i $(i=1, 2,\cdots)$, there exists $x=\bigcup_{i=1}^{\infty} x_i$.
3) $M\subset B$ is an ideal (in Birkhoff's terminology [1]) if $a\in M$, $b\leqq a$ implies $b\in M$.
4) Complement of σ-open set.