22. *An Abstract Analyticity in Time for Solutions of a Diffusion Equation*

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1. *Introduction and the result.* Consider an equation of evolution

$$\frac{\partial u}{\partial t}=Au, \qquad t>0, \tag{1.1}$$

where the differential operator

$$A=a^{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j}+b^i(x)\frac{\partial}{\partial x_i}+c(x) \tag{1.2}$$

is elliptic in a connected domain G of an m-dimensional euclidean space E^m. Under certain conditions upon the coefficients a, b and c of A, we can specify a linear subspace D of $L_2(G)$ with the following three properties.

(i) The functions $\in D$ are C^∞ in G, and D is $L_2(G)$-dense in $L_2(G)$ such that $Af \in L_2(G)$ for $f \in D$.

(ii) If we consider A as an operator on $D \subseteqq L_2(G)$ into $L_2(G)$, then A admits, in $L_2(G)$, the smallest closed extension $\hat{A}$.

(iii) $\hat{A}$ is the infinitesimal generator of a semi-group T_t of normal type in $L_2(G)$ such that, for any $f \in L_2(G)$, $u(t,x)=(T_t f)(x)$ is a solution of (1.1) with the initial condition

$$L_2(G)\text{-}\lim_{t \downarrow 0} u(t,x)=f(x) \tag{1.1$'$}$$

satisfying the "forward and backward unique continuation property":

(1.3) If, for a fixed $t_0>0$, $u(t_0,x)\equiv 0$ on an open set $G_0 \subseteqq G$, then $u(t,x)=0$ for every $t>0$ and every $x \in G_0$.

The proof of (1.3) is based upon the fact that $T_t f$ is an $L_2(G)$-valued abstract analytic function of t in a certain sector of the complex plane which contains the positive t-axis in its interior and with $t=0$ as its vertex. Such abstract analyticity in time is implied by the estimate (2.11) below of the resolvent of $\hat{A}$.[1]

Our result (1.3) gives a partial answer to a conjecture proposed by S. Ito and H. Yamabe [2]. Actually, our solution $u(t,x)=(T_t f)(x)$ enjoys the "unique continuation property":

(1.3)$'$ If, for a fixed $t_0>0$, $u(t_0,x)\equiv 0$ on an open set $G_0 \subseteqq G$, then $u(t,x)=0$ for every $t>0$ and every $x \in G$.

1) This estimate was given in the author's lecture at Yale University in the fall of 1958.