52. On Locally Q-complete Spaces. I

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If Z is a Q-space containing X as a dense subspace, then we call Z a *Q-completion of X*.[1] X is said to be *locally complete with respect to the structure*[2] μ of X if for any point $x \in X$, there is a neighborhood whose closure is complete with respect to μ. If μ is the structure generated by $C(X)$,[2] then we say that X is *locally Q-complete*. X is called a local Q-space if for any point $x \in X$, there is a neighborhood of x whose closure is a Q-space. It is obvious that any Q-space is locally Q-complete and any locally Q-complete space is a local Q-space. If X is normal and is a local Q-space, then X is locally Q-complete.

In this paper, we shall establish some relations between a locally Q-complete space and its Q-completion, which are analogous to the relations between a locally compact space and its compactification.

Lemma 1. *Let B be a closed subset of X and Z a space obtained from X by contracting B to a point. If either X is normal*[3] *or B is compact, then Z is completely regular.*

This lemma is easily proved by the normality of X or the compactness of B respectively. In general, the space Z mentioned above is not necessarily completely regular.

Lemma 2. *Let Y be a Q-space and F a closed subset of Y, and Z be a space obtained from Y by contracting F to a point p in Y. If Z is completely regular, then $X=Z-\{p\}$ is locally complete with respect to the structure generated by $C_0=\{f; f \in C(Z),\ f(p)=0\}$.*

Proof. We notice first that C_0 is considered as a subring C_1 of $C(Y)$ whose elements vanish at every point of $B=F \cup \{p\}$. For any point x in X, there is a neighborhood V such that $\overline{V}(\text{in } Z) \not\ni p$ in Z. To prove that $\overline{V}(\text{in } Z)$ is complete with respect to the structure generated by C_0, it is sufficient to prove that $U=\overline{V}(\text{in } Z)$, considered as a closed subset of Y, is complete with respect to the structure μ

1) A space considered here is always a completely regular T_1-space. $C(X)$ denotes the totality consisting of all real-valued continuous functions defined on X, and $B(X)$ denotes a subset of $C(X)$ consisting of all bounded functions.

2) A *structure* of X considered here means a uniformity of X which agrees the given topology of X. A *structure generated by C*, which is a subset of $C(X)$, is a structure given by the following uniform neighborhoods: $W(x; f_1, \cdots, f_n, \varepsilon)=l\{y; |f_i(x)-f_i(y)|<\varepsilon\}$ where $f_i \in C$ and ε is an arbitrary positive real number.

3) In case X is normal, Z is normal.