49. *On the Extensions of Finite Factors. II*

By Zirô TAKEDA
Ibaragi University

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Since extensions of a continuous finite factor A are closely related with extensions of the group K of all inner automorphisms of A [2], some fundamentals of the cohomology theory of groups reflect upon constructions of extended factors. In this paper we shall show that the effectiveness of a group G of automorphism classes for the construction of extended factors is decided by the fact that a three-dimensional cochain associated with G is coboundary or not. In general, the group K has no central element other than 1 and so, by a proposition of group extensions, the extension of K by G is uniquely determined within equivalences. On the other hand we shall define an equivalence relation in factors extended by G analogously to the one for extended groups and then show that the equivalent classes of extensions of A by G are one-to-one correspondent to the second cohomology group $H^2(G, Z)$, where Z is the unit circle in the complex plane and G is assumed to act on Z trivially.

1. We use the same notations as in [2] as possible. By A we mean a continuous finite factor acting on a separable Hilbert space and by $\tilde{\mathfrak{A}}$ the group of all $*$-automorphisms of A. Denote by K the group of all inner automorphisms of A. K is a normal subgroup of $\tilde{\mathfrak{A}}$. Put $\mathfrak{A}$ the quotient group $\tilde{\mathfrak{A}}/K$. We take up an enumerable subgroup G of $\mathfrak{A}$. We call G *a group of automorphism classes.* For every element $\alpha \in G$ we choice a representative $\bar{\alpha}$ in the coset α of the quotient $\tilde{\mathfrak{A}}/K$, then for every α and β there occurs $m_{\alpha,\beta} \in K$ such that $\bar{\alpha}\cdot\bar{\beta}=\overline{\alpha\beta}\cdot m_{\alpha,\beta}$. This satisfies relations:

$$(1) \qquad (k^{\alpha})^{\beta} = (k^{\alpha\beta})^{m_{\alpha,\beta}} \qquad \text{for } k \in K$$

$$(2) \qquad m_{\alpha,\beta\gamma} m_{\beta,\gamma} = m_{\alpha\beta,\gamma} m^{\gamma}_{\alpha,\beta}$$

where $k^{\alpha}=\bar{\alpha}^{-1}k\bar{\alpha}$ and $k^{m}=m^{-1}km$ $(m \in K)$. We call such a system $\{m_{\alpha,\beta}\}$ *a factor set of inner automorphisms of A.* If a factor set $\{m_{\alpha,\beta}\}$ satisfies $m_{\alpha,\alpha^{-1}}=1$ for every α, it is *normalized.* In this paper we consider only such a group for which normalized factor sets exist. For a factor set $\{m_{\alpha,\beta}\}$, we get an extension $\boldsymbol{K}$ of the group K by G, which we show by $\boldsymbol{K}=(K, G, m_{\alpha,\beta})$ [1, 2].

Let $\boldsymbol{K}^{(1)}=(K, G, m^{(1)}_{\alpha,\beta})$ and $\boldsymbol{K}^{(2)}=(K, G, m^{(2)}_{\alpha,\beta})$ be two extensions of a group K by a group G with respect to different factor sets $\{m^{(1)}_{\alpha,\beta}\}$ and $\{m^{(2)}_{\alpha,\beta}\}$ respectively. If there is an isomorphism between $\boldsymbol{K}^{(1)}$ and $\boldsymbol{K}^{(2)}$ satisfying