45. *Note on a Theorem for Dimension*

By Jingoro SUZUKI
Department of Mathematics, Tokyo University of Education, Tokyo
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1. Recently, K. Nagami has proved the following theorem [4]:

Let X and Y be metric spaces and f a closed continuous mapping of X onto Y. If $f^{-1}(y)$ consists of exactly $k(<\infty)$ points for every point $y \in Y$ and dim $X \leqq 0$, then we have dim $Y \leqq 0$.

In the present note, as an extension of this theorem, we shall prove the following theorem:

Theorem. *Let f be a closed continuous mapping of a metric space X onto a topological space Y such that for each point y of Y the inverse image $f^{-1}(y)$ consists of exactly $k(<\infty)$* points, *then we have*

$$\dim X = \dim Y.$$

To prove the theorem, we use some lemmas:

Lemma 1 (K. Morita [2]). *In order that a T_1-space X be metrizable it is necessary and sufficient that there exist a countable collection $\{\mathfrak{F}_j\}$ of locally finite closed covering of X satisfying the condition:*

For any neighborhood U of any point x of X there exists some j such that $S(x, \mathfrak{F}_j) \subset U$.

Lemma 2 (K. Morita and S. Hanai [3], A. H. Stone [5]). *Let f be a closed continuous mapping of a metric space X onto a topological space Y. In order that Y be metrizable it is necessary and sufficient that the boundary $\mathfrak{B}f^{-1}(y)$ of the inverse image $f^{-1}(y)$ be compact for every point y of Y.*

2. **Proof of the theorem.** Let us put $f^{-1}(y)=\{x_i(y) \mid i=1, 2, \cdots, k\}$ for every point y of Y. By Lemma 1 there exist a countable number $\{\mathfrak{F}_j\}$ of locally finite closed coverings of X such that for some integers j_i and some indices $\alpha_i \in \Omega_{j_i}$ we have

$$F_{j_i\alpha_i} \ni x_i(y), \qquad i=1, 2, \cdots, k$$

and

$$F_{j_i\alpha_i} \frown F_{j_l\alpha_l} = \phi, \qquad i, j=1, 2, \cdots, k, \qquad i \neq l,$$

where we put $\mathfrak{F}_j=\{F_{j\alpha} \mid \alpha \in \Omega_j\}$, $\quad j=1, 2, \cdots$.

Let us put $\displaystyle\bigcap_{i=1}^{k} f(F_{j_i\alpha_i}) = W_y$. As f is a closed mapping, W_y is a closed subset of Y and contains y. If we denote by f_1 the partial mapping f whose domain is $F_{j_1\alpha_1} \frown f^{-1}(W_y)$, and whose range is W_y, then f_1 is a homeomorphism from $F_{j_1\alpha_1} \frown f^{-1}(W_y)$ onto W_y. Hence we have