72. *On the Singular Integrals.* VI*)

By Sumiyuki KOIZUMI
Department of Mathematics, Hokkaidô University, Sapporo, Japan
(Comm. by Z. SUETUNA, M.J.A., July 13, 1959)

1. We begin with the following

Definition 1. *By W_2 we denote the class of functions which are measurable over $(-\infty, \infty)$ and satisfy*

$$\int_{-\infty}^{\infty} \frac{|f(t)|^2}{1+t^2} dt < \infty. \tag{1.01}$$

For this class, the generalized Hilbert transform of order 1 is precisely corresponding. This modified one is defined as follows [4, V]:

$$\tilde{f}_1(x) = \frac{x+i}{\pi} \int_{-\infty}^{\infty} \frac{f(t)}{t+i} \frac{dt}{x-t}. \tag{1.02}$$

The main purpose of this chapter is to determine the relation of spectrum between any given function $f(x)$ of the class W_2 and its generalized Hilbert transform of order 1. We shall quote the Plancherel theorem of Fourier transform repeatedly [2]. We introduce the generalized Fourier transform due to N. Wiener [6]. This is defined by

$$\begin{aligned} s^f(u) = & \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} f(x) \frac{e^{-iux}-1}{-ix} dx \\ & + \underset{A\to\infty}{\text{l.i.m.}} \frac{1}{\sqrt{2\pi}} \left[\int_{-A}^{-1} + \int_{1}^{A}\right] f(x) \frac{e^{-iux}}{-ix} dx. \end{aligned} \tag{1.03}$$

Then by the Plancherel theorem, the Fourier-Wiener transform $s^f(u)$ is well defined and

$$s^f(u+\varepsilon) - s^f(u-\varepsilon) = \underset{A\to\infty}{\text{l.i.m.}} \frac{1}{\sqrt{2\pi}} \int_{-A}^{A} f(t) \frac{2\sin \varepsilon t}{t} e^{-iut} dt, \tag{1.04}$$

$$\frac{1}{4\pi\varepsilon} \int_{-\infty}^{\infty} |s^f(u+\varepsilon) - s^f(u-\varepsilon)|^2 du = \frac{1}{\pi\varepsilon} \int_{-\infty}^{\infty} |f(t)|^2 \frac{\sin^2 \varepsilon t}{t^2} dt. \tag{1.05}$$

If $f(x)$ belongs to the class W_2, then by Theorem 1 of [4, V] the Fourier-Wiener transform of $\tilde{f}_1(x)$ is also defined. We will denote this by $\tilde{s}^f_1(u)$.

Throughout this paper, let $g(x)$ be a real valued measurable function which belongs to the class W_2. We also denote

$$f_1(x) = g(x) + i\tilde{g}_1(x). \tag{1.06}$$

We shall prove the following fundamental

Theorem 1. *Let $g(x)$ belong to the class W_2. Then for any given positive number ε,*

---

*) Details will appear in Jour. Fac. Sci., Hokkaidô Univ.