101. *Purely Algebraic Characterization of Quasiconformality*

By Mitsuru NAKAI
Mathematical Institute, Nagoya University
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1. Consider two Riemann surfaces R and R'. Assume the existence of a quasiconformal mapping[1] T of R onto R' in the sense of Pfluger-Ahlfors-Mori [6,1,3]. In this case we say that R and R' are *quasi-conformally equivalent*. In particular, if the maximal dilatation $K(T)$ (of T (cf. [1]) is 1), R and R' are said to be *conformally equivalent*.

This note will communicate a certain criterion of quasiconformal equivalence in terms of function algebras, details of which will be published later.

2. Let R be a Riemann surface and $M(R)$ be *Royden's algebra* [8, 4] associated with R, i.e. the totality of complex-valued bounded a.c.T.[2] functions on R with finite Dirichlet integrals over R. The algebraic operations are defined as follows: $(f+g)(p)=f(p)+g(p)$, $(f\cdot g)(p)=f(p)\cdot g(p)$ and $(\alpha\cdot f)(p)=\alpha f(p)$. Then $M(R)$ is a commutative algebra over the complex number field.

3. As an improvement of the author's previous result [4], we mention the following algebraic criterion of quasiconformal equivalence:

Theorem 1. *Two Riemann surfaces R and R' are quasi-conformally equivalent if and only if $M(R)$ and $M(R')$ are algebraically isomorphic.*

4. Royden's algebra $M(R)$ can be *normed* by the following:

$$\|f\|=\sup_R|f|+\left(\iint_R df\wedge{}^*\overline{df}\right)^{1/2}.$$

As a special case of Theorem 1 and as an improvement of [5], we get the following normed algebraic criterion of conformal equivalence:

Theorem 2. *Two Riemann surfaces R and R' are conformally equivalent if and only if $M(R)$ and $M(R')$ are isometrically isomorphic.*

5. Theorems 1 and 2 follow from the following more precise facts.

Let $Q(R,R')$ be the totality of quasiconformal mappings of R onto R' and $I(R,R')$ be the totality of algebraic isomorphism of $M(R)$ onto $M(R')$. Then there exists a one-to-one correspondence $T\leftrightarrow\sigma$ between $Q(R,R')$ and $I(R,R')$. This correspondence is given by $f^\sigma=f\circ T^{-1}$

1) Including both direct and indirect ones.

2) Abbreviation of "absolutely continuous in the sense of Tonelli". For the definition, refer to [7, 9, 10].