# 100. *Finite-to-one Closed Mappings and Dimension. II*

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In this note[1] our concern is devoted to mappings defined on spaces of positive dimension, though in the previous note [3] we were mainly concerned with mappings defined on 0-dimensional spaces. Theorem 1 below gives an answer for the problem concerning dimension-raising mappings between non-separable metric spaces, which was raised by W. Hurewicz [1] and solved for the case of separable metric spaces by J. H. Roberts [4]. All notations and terminologies used here are the same as in the previous note [3]. A space $R$ has dimension $\leq \aleph_0$, $\dim R \leq \aleph_0$, if $R$ is the countable sum of subspaces $R_i$ with $\dim R_i \leq 0$.

Let $R$ and $S$ be topological spaces. Let $\mathfrak{F}=\{F_\alpha; \alpha \in A\}$ and $\mathfrak{H}=\{H_\alpha; \alpha \in A\}$ be respectively locally finite closed coverings of $R$ and $S$. Let $f$ be a mapping of $R$ onto $S$. Let $r$ be a positive integer. If the following conditions are satisfied, $(R, \mathfrak{F}, f)$ is called *the cut of order* $r$ of $(S, \mathfrak{H})$.

(1) For every $\alpha \in A$, $f|F_\alpha$ is a homeomorphism of $F_\alpha$ onto $H_\alpha$.

(2) If order $(y, \mathfrak{H})$, the number of closed sets of $\mathfrak{H}$ which contain $y \in S$, is greater than $r$, $f^{-1}(y)$ consists of one and only one point. If $r_1=$order $(y, \mathfrak{H})$ is not greater than $r$, $f^{-1}(y)$ consists of exactly $r_1$ points.

$R$ is called the cut-space of order $r$ obtained from $(S, \mathfrak{H})$. $\mathfrak{F}$ is called the derived covering of order $r$ and $f$ the cut-mapping. We can prove that there exists the cut of order $r$ of $(S, \mathfrak{H})$ for any $(S, \mathfrak{H})$ and $r$ and that the cut is essentially unique.

Let $R_0$ be a metric space with $\dim R_0=n$, $0<n<\infty$. Let $m$ be an arbitrary integer with $0 \leq m < n$. We shall now construct a metric space $T$ with $\dim T=m$ and a closed mapping $\pi_0$ of $T$ onto $R_0$ such that for every point $p$ of $R_0$ $\pi_0^{-1}(p)$ consists of at most $n-m+1$ points.

By [2] or [3] there exist $\lim A_i=\lim \{A_i, f_{i+1, i}\}$, where $A_i$ are discrete spaces of indices, and a sequence of locally finite closed coverings $\mathfrak{F}_{0i}=\{F(0, \alpha_i); \alpha_i \in A_i\}$, $i=1, 2, \cdots$, which satisfy the following conditions.

(1) The diameter of each set of $\mathfrak{F}_{0i}<1/i$.

(2) The order of every $\mathfrak{F}_{0i} \leq n+1$.

1) The detail of the content of the present note will be published in another place.