# 98. On Locally Q-complete Spaces. III

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(Comm. by K. KUNUGI, M.J.A., Oct. 12, 1959)

We assume always that $X^{*)}$ is locally $Q$-complete but not a $Q$-space. Then there are one-point $Q$-completions of $X$ [2]. In this paper, we shall investigate some properties of one-point $Q$-completions of $X$. We noticed, in [2], that $X$ is open in $\nu X$ and $X\smile(\nu X-X)^{\beta}$ is a $Q$-space. We have similarly that if $B$ is any compact subset in $\beta X-X$ which contains $\nu X-X$ then the space $X\smile B$ is also a $Q$-space, and moreover the space $Z$ obtained from $X\smile B$ by contracting $B$ to a point in $B$ is a one-point $Q$-completion (Theorem 1 in [2]). In the following, we shall prove that any one-point $Q$-completion of $X$ is given as an image of a space $X\smile B$ under a continuous mapping $\varphi$ such that $\varphi|X$ is a homeomorphism which leaves every point of $X$ invariant where $B$ is some compact subset in $\beta X-X$ which contains $\nu X-X$.

**Lemma 1.** *Suppose that $Z=X\smile\{p\}$ is a one-point $Q$-completion of $X$. Then there is a continuous mapping $\psi$ of $\nu X$ onto $Z$ such that $\psi(\nu X-X)=\{p\}$, $\psi(x)=x$ for every $x\in X$ and $\psi|X$ is a homeomorphism.*

*Proof.* $X$ is considered as a uniform space $X_1$ with the structure generated by $C=\{f|X;\ f\in C(Z)\}$ and $Z$ becomes a completion of $X_1$. On the other hand, $X$ may be considered as a uniform space $X_2$ with the structure generated by $C(X)$. Since $C(X)\supset C$ and the identical mapping $i$ is uniformly continuous, $i$ has a continuous extension $\psi$ of $\nu X$ to $Z$. Hence, to prove Lemma, it is sufficient to show that $\psi(\nu X-X)=p$. Suppose that $\{a_\alpha;\ a_\alpha\in X\}\to a\in\nu X-X$ and $\psi(a)=b\in X\subset Z$. We take an open neighborhood $V$(in $Z$) of $b$ which does not contain $p$. $i^{-1}(V)$ is open in $\nu X$ because $X$ is open in $\nu X$. By the assumption, for some index $\alpha_0$, $\alpha>\alpha_0$ implies $\psi(a_\alpha)=i(a_\alpha)\in V$, and hence $i^{-1}(V)\ni a_\alpha$ for $\alpha>\alpha_0$. This is a contradiction. We have therefore that $\psi(\nu X-X)=p$.

For any point $x\in Z$, let us put $B(x)=\frown\overline{\psi^{-1}(V)}$ (in $\beta X$) where $V$ runs over all neighborhoods (in $Z$) of $x$. Since $\psi(\nu X-X)=p$. $B(p)$ is a compact subset containing $\nu X-X$.

**Lemma 2.** *$B(x)=\{x\}$ for any $x\in X\subset Z$ and $B(p)\subset\beta X-X$.*

*Proof.* For any point $y\in X\subset Z$, there is an open neighborhood $U$ (in $Z$) of $y\in X\subset Z$ which is disjoint from some neighborhood (in $Z$) of $p$. We have therefore $B(p)\not\ni y$, which implies that $B(p)\subset\beta X-X$. Simi-

*) A space $X$ considered here is always a completely regular $T_1$-space, and other terminologies used here, for instance "$Q$-completion," are the same as in [2,3].